

Treatment Designs with Nested or Nested and Crossed Factors

Designs with nested factors, in which the levels of one factor (B) are not identical across all levels of another factor (A) occur commonly in many types of research. The model for a nested design with a fixed factor A, and a random nested factor, B within A is:

$$y_{ijk} = \mu + \alpha_i + b_{j(i)} + e_{k(ij)},$$

where every random term is assumed independent of the others and normally distributed with mean 0 and its own variance component. In the model above the A factor is fixed, but it is not uncommon for it to be a random effect. As with other designs we use the information from the expected mean squares for each effect to determine which F ratios to use to test hypotheses. The method of moments approach is used to present estimators of variance components. The use of staggered nested designs to yield more balanced degrees of freedom across effects is also presented.

Some experimental designs use both crossed and nested effects, often called a nested factorial design. One example would be if we had a fixed factor A, a random factor B crossed with A, a random factor C nested within B, with replicate samples within the nested factor. For this example the model would be:

$$y_{ijkl} = \mu + \alpha_i + b_j + c_{k(j)} + (ab)_{ij} + (ac)_{ik(j)} + e_{l(ijk)},$$

where the b_j , $c_{k(j)}$, $(ab)_{ij}$, $(ac)_{ik(j)}$, and $e_{l(ijk)}$ terms are assumed independent and normally distributed with 0 means and their own variance components. Here too, the analyses must take account of expected mean squares. See the SAS and R code for examples of analyzing data from these designs.