Randomized Complete Block Designs and Latin Square Designs

As we have seen in previous examples, in many cases there is variation in a response due to a factor other than the experimental treatment. Taste test examples are always good illustrations of this idea. In a randomized complete block design, we attempt to control this extraneous variation by applying each level of the treatment to homogeneous units called blocks. Blocks could be people, contiguous areas in a field, etc. The RCB design uses a restricted randomization, in which the set of treatment levels is separately randomized in each block so that each block receives each treatment level once. The linear model for an RCB is:

$$y_{ij} = \mu + \alpha_i + \beta_j + e_{ij},$$

where the treatment and block effects are assumed to be additive, meaning that they do not interact. As the text mentions, the analysis of data from a RCB is the same as for a two-factor factorial design (CRF) that does not have replication. Some issues that arise with RCB designs:

The additivity assumption

The text discusses Tukey's one-degree of freedom test for nonadditivity in Chapter 9. The approach for this test described in the text also suggests a power transformation for use when nonadditivity exists.

Related RCB designs

If we have multiple experimental units per treatment per block, then we can conduct traditional tests for interaction, this design is sometimes called a generalized randomized complete block design. We can also have factorial structure for the treatments.

The usual assumptions, did the blocking work?

The text discusses the use of residual plots as discussed in previous chapters to check the usual assumptions on the errors. One way to assess the use of blocking is through the concept of relative efficiency. The relative efficiency of the RCB design to a completely randomized (CRD) design is:

$$\text{corrected } RE = \frac{(\nu_{rcb}+1)(\nu_{crd}+3)}{(\nu_{rcb}+3)(\nu_{crd}+1)} \frac{s^2_{crd}}{s^2_{rcb}}$$

where s^2_{rcb} is the mean squared error from the RCB analysis, ν_{rcb} is the error degrees of freedom for the RCB design, ν_{crd} is the error degrees of freedom if the data had come from a CR design, and s^2_{crd} is:

$$s^2_{crd} = \frac{(r-1)MS_{\text{Blocks}} + ((g-1)+(r-1)(g-1))MS_E}{(r-1)+(g-1)+(r-1)(g-1)}$$

The ratio MSB/MSE from the RCB analysis can also be useful as a quick guide to the usefulness of blocking.

Random blocks

In many applications it is reasonable to assume that the blocks are a random effect. In these cases the ANOVA test for H_0 for the fixed effect does not change, and the standard error of the differences between means does not change, but the standard error of individual treatment means does change.

Latin Square Designs

Latin Square designs are used to achieve error reduction when there are two blocking variables, and both of them and the treatment factor have the same number of levels. The restriction to have the same number of levels makes Latin square designs less useful for a small number of treatment levels, and fairly costly for a large number of treatment levels. The layout of a typical design is shown, and the procedure to randomize a Latin square design is mentioned in the text. The model for a Latin square design is:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + e_{ijk}$$

with all three terms having g levels and the usual assumptions on the error terms e_{ijk}. Many of the same topics discussed for RCB designs are discussed, such as a test for nonadditivity, and calculation of design efficiencies. The idea of having replicated Latin squares is discussed to help with error degrees of freedom when the number of treatment levels is small.