Fractional Factorial Designs

In Chapter 15 we learned how to sacrifice information on certain effects in a factorial experiment to enable us to test other effects, using smaller blocks than in a randomized block factorial design. In Chapter 18 we will use these same ideas to allow us to design experiments to investigate k factors without using all 2^k treatment combinations.

In our confounded 2^3 block design from the chapter 15, we divided the eight combinations into two groups of 2^{k-1} or four combinations based on the signs of the ABC interaction. Similarly, we can obtain a half fraction of a 2^k factorial experiment by only using the half of the treatment levels that correspond to the same sign of a chosen effect, which is typically the highest order interaction. Thus, for a 2^3 factorial experiment we would typically use the ABC interaction and only use the treatment combinations all having a '+' for the ABC effect. Here we call ABC the **design generator**, and if we use the '+' levels of ABC we would say that $I = ABC$ is the **defining relation**. If we use the '-' levels of ABC then $I = -ABC$ is the defining relation.

In a fractional factorial experiment by not using all combinations of all factors, we introduce **aliasing** between factors. The aliases for a given effect can be determined from their generalized interaction with the defining relation of the design. For a half fraction of a design we would use the highest order interaction as the defining relation, note that in that case the design is a complete factorial for the other factors in the design.

The concept of **resolution** is explained in the text, and is important for understanding the amount of information that is aliased in a fractional factorial design. The resolution is determined by the smallest number of characters in the design generator. A notation is introduced to state the resolution of a design in subscripted Roman letters.

The text also illustrates a way to analyze data from an unreplicated half fraction of a 2^k factorial experiment using a normal probability plot of effects. This display can be followed by a subjective step to pool supposedly inactive effects into error to allow tests of the active effects. The text then shows how to develop quarter fractions of 2^k factorial experiments by using two defining generators. The two generators and their generalized interaction then specify the defining relation for the quarter fraction design. In a quarter fraction of a 2^k factorial experiment each effect has three aliases, which can be determined from the defining relation.