1 Power and sample size for completely randomized designs

Power and sample size analyses are important tools for assessing the ability of a statistical test to detect when a null hypothesis is false, and for deciding what sample size is required for having a reasonable chance to reject a false null hypothesis.

Recall that for a test of a statistical hypothesis, the Type I error (α) is the probability of rejecting the null hypothesis when it is true. The Type II error (β) is the probability of not rejecting the null hypothesis when it is false [Note: out text uses the symbols like ε_I and ε_{II} to denote Type I and II errors, respectively]. The power of the test equals $1-\beta$, and is the probability of rejecting the null hypothesis when it is false. The power will depend on the alternative hypothesis, and we would like to have high power to detect alternative hypotheses of interest.

For the completely randomized design ($y_{ij} = \mu + \alpha_i + e_{ij}$) with equal sample size per group n, when the null hypothesis is true, the F statistic has an F distribution with $g-1$ and $g(n-1)$ degrees of freedom. When the null hypothesis is false, the F statistic follows a non-central F distribution with $g-1$ and $g(n-1)$ degrees of freedom, and noncentrality parameter:

$$\varsigma = \frac{\sum n_i \alpha_i^2}{\sigma^2} = \frac{n \sum \alpha_i^2}{\sigma^2}.\text{(Note: the symbol } \lambda \text{ is more commonly used)}$$

The power of the F test is a monotonically increasing function of the parameter ς. Notice that when the null hypothesis is true, $\varsigma = 0$, so that the usual (or central) F distribution is just a special case of the non-central F distribution. It should make sense intuitively that the power of the F test increases as n increases, as $\sum \alpha_i^2$ increases, and as σ^2 decreases, as predicted by the noncentrality parameter ς.

Note: some texts also introduce a related measure: $\Phi = \sqrt{\frac{\varsigma}{g}}$.We will show how to use SAS and R to do our power and sample size calculations.