1 Further topics in random and mixed effects models

1.1 The Intraclass Correlation Coefficient

One measure of interest in a random effects model is the ratio of the variance of the random effect to the total variance of the response. This quantity is called the intraclass correlation coefficient:

$$\rho_I = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}$$

The intraclass coefficient ρ_I is a measure of similarity of the observations within the same treatment group level. We will use maximum-likelihood-based estimators of the variance components, as shown in our code.

1.2 Power and sample size for the random effects model

For the completely randomized design with random effects and one-way treatment structure, when the null hypothesis is true, the F statistic has an F distribution with $g - 1$ and $g(n - 1)$ degrees of freedom. When the null hypothesis is false, the distribution of the F statistic is a multiple of a central F distribution with $g-1$ and $g(n-1)$ degrees of freedom, where the multiplicative constant is equal to:

$$\lambda = 1 + \frac{n\sigma_a^2}{\sigma_e^2},$$

so that we can calculate the power by

$$P\left[F > \frac{1}{\lambda}F_{\alpha,\nu_1,\nu_2}\right],$$

where λ is the value of the multiplicative constant under H_a and F_{α,ν_1,ν_2} is a tabled F value for $\alpha = $ Type I error, and degrees of freedom ν_1 and ν_2. Power can then be calculated by specifying a value for the ratio σ_a^2/σ_e^2, using similar programs to what we used for fixed effects.

1.3 Restricted vs. Unrestricted Mixed models

The text has an extensive discussion of the comparison of the restricted versus unrestricted mixed model. The issue has to do with how we think about interactions of fixed and random effects, and if there are constraints on the sum of the interaction effects across the fixed factor. The author of our text makes many good points and prefers the restricted model. Most software assumes the unrestricted model so we will generally assume that model.