Statistics 550 Homework assignment 2

Problems 1-3 are problems 8.10, 9.6, and 9.15 in the text, respectively.

4. For the simple linear regression model, $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, verify that the vector partial derivatives of the sum of squares function $S(\mathbf{b})$ are correct. In other words, write out the scalar expression for $\mathbf{y}'\mathbf{y} - (2\mathbf{y}'\mathbf{X})\mathbf{b} + \mathbf{b}'(\mathbf{X}'\mathbf{X})\mathbf{b}$ and then verify that the partial derivative equations can be rewritten in the form $-2\mathbf{X}'\mathbf{y}+2\mathbf{X}'\mathbf{X}\mathbf{b}$.

5. For a small data set in which two values (12 and 36) are recorded for group 1 and two values (54 and 72) are recorded for group 2, consider the model

$$y_{ij} = \mu + \alpha_j + \epsilon_{ij},$$

where deviation coding is used for the α_j parameter (use a full rank approach, so there is just one α_j parameter). Write down the $\mathbf{X}$ matrix for the model and then use the matrix-based approach to calculate the least-squares estimates $\widehat{\boldsymbol{\beta}}$ and their sampling variances $\widehat{V}(\widehat{\boldsymbol{\beta}})$ by hand (for estimating $\widehat{V}(\widehat{\boldsymbol{\beta}})$, use the fact that $\widehat{\sigma}_{\varepsilon}^2 = 225$.). Specify a matrix $\boldsymbol{L}$ and a vector $\mathbf{c}$ so that the null hypothesis of no difference between groups can be tested with the F statistic:

$$F_0 = \frac{(\boldsymbol{L}\mathbf{b} - \mathbf{c})'[\boldsymbol{L}(\mathbf{X}'\mathbf{X})^{-1}\boldsymbol{L}']^{-1}(\boldsymbol{L}\mathbf{b} - \mathbf{c})}{qS_E^2},$$

then compute the F_0 statistic by hand. Verify your results by comparing them to the output from a computer regression analysis.