The Matrix-Based Approach to the General Linear Model - continued still further

Joint Confidence Regions

The F test given above for a subset of regression coefficients can be inverted to obtain a joint confidence region:

$$\Pr\left[\frac{(\mathbf{b}_1 - \boldsymbol{\beta}_1^{(0)})'\mathbf{V}_{11}^{-1}((\mathbf{b}_1 - \boldsymbol{\beta}_1^{(0)}))}{qS_E^2} \leq F_{\alpha,q,n-k-1}\right] = 1 - \alpha.$$

This region can be described as the set of all $\boldsymbol{\beta}_1$ values for which $(\mathbf{b}_1 - \boldsymbol{\beta}_1^{(0)})'\mathbf{V}_{11}^{-1}((\mathbf{b}_1 - \boldsymbol{\beta}_1^{(0)})) \leq qS_E^2 F_{\alpha,q,n-k-1}$. This is in general an ellipsoidal region that will become spherical if the covariance of the $\mathbf{b}_1$ estimates is 0. The use of this joint confidence region multiplier $qS_E^2 F_{\alpha,q,n-k-1}$ for individual coefficients is the motivation behind Scheffe's multiple comparison method for general contrasts. The text illustrates the distinction between joint and individual confidence regions for the special case of two dimensions ($q = 2$), including the different effect of covariance between the $\mathbf{X}$ variables on the data ellipse and the confidence region ellipse.

Multivariate Linear Models

The text has an introduction to the multivariate linear model and presents the maximum likelihood estimator under the assumption of independent multivariate normal rows of the error matrix. There are multiple test statistics to consider in the multivariate case, which are introduced.