Some Matrix and Statistical Results, Part 1

Starting with Chapter 9 in the text we will cast linear models in matrix notation. The results in the text are fairly self-contained, but we will use some results that are not in the text, and it will be useful to have some results not found in our text.

Definition: The rank of a matrix, rank(A), is the number of linearly independent columns in the matrix.

Result 1:

i) The number of linearly independent rows of a matrix is the same as the number of linearly independent columns, hence is also equal to the rank of the matrix.

ii) For a nonzero matrix A of size n x p, the rank of the matrix satisfies $1 \leq \text{rank}(A) \leq \min(n, p)$.

iii) If two matrices A and B are conformable, then $\text{rank}(AB) \leq \min(\text{rank}(A),\text{rank}(B))$

iv) $\text{rank}(A) = \text{rank}(AA') = \text{rank}(A'A)$

Result 2: Let $\mathbf{y}$ be a vector random variable of length n, with mean vector $E(\mathbf{y}) = \boldsymbol{\mu}$ and covariance matrix $V(\mathbf{y}) = \boldsymbol{\Sigma}$, and $\mathbf{Y}$ a random matrix of size $n\,\mathbf{x}\,p$. Let $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ be matrices of constants. Then if appropriate matrices are conformable the following are true:

i) $E(\mathbf{Ay}) = \mathbf{A}E(\mathbf{y}) = \mathbf{A}\boldsymbol{\mu}$,

ii) $V(\mathbf{Ay}) = \mathbf{A\Sigma A'}$

iii) $E(\mathbf{AYB} + \mathbf{C}) = \mathbf{A}E(\mathbf{Y})\mathbf{B} + \mathbf{C}$

iv) $E(\mathbf{y'Ay}) = trace(\mathbf{A\Sigma}) + \boldsymbol{\mu}'\mathbf{A}\boldsymbol{\mu}$

References

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Sen, A., and Srivastava, M. 1990. Regression Analysis: Theory, Methods, and Applications, New York: Springer-Verlag, Inc.