

## Some More Matrix and Statistical Results

**Definition:** If  $\mathbf{y}$  is a vector random variable of length  $n$  with mean vector  $E(\mathbf{y}) = \boldsymbol{\mu}$ , covariance matrix  $V(\mathbf{y}) = \boldsymbol{\Sigma}$ , and has a probability density function of:

$$f(\mathbf{y}) = |2\pi\boldsymbol{\Sigma}|^{-1/2} \exp\left\{-\frac{1}{2}(\mathbf{y}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{y}-\boldsymbol{\mu})\right\},$$

then  $\mathbf{y}$  has a multivariate normal distribution, denoted by  $\mathbf{y} \sim N_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ .

Note that this is a slightly more general version than shown on page 197 of the text, since here  $V(\mathbf{y}) = \boldsymbol{\Sigma}$  whereas on that page they assume  $V(\mathbf{y}) = \sigma_\varepsilon^2 \mathbf{I}_n$ .

**Result 3:** Let  $\mathbf{y} \sim N_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ , let  $\mathbf{C}$  be an  $m \times n$  matrix, and  $\mathbf{d}$  be an  $m \times 1$  vector. Then  $\mathbf{C}\mathbf{y} + \mathbf{d} \sim N_m(\mathbf{C}\boldsymbol{\mu} + \mathbf{d}, \mathbf{C}\boldsymbol{\Sigma}\mathbf{C}')$ .

### References

Hocking, R.R. 1996. *Methods and Applications of Linear Models: Regression and the Analysis of Variance*, New York: John Wiley & Sons, Inc.

Seber, G.A.F., and Lee, A.J. 2003. *Linear Regression Analysis*, Second Edition, Hoboken, NJ: Wiley-Interscience.

Sen, A., and Srivastava, M. 1990. *Regression Analysis: Theory, Methods, and Applications*, New York: Springer-Verlag, Inc.