The Linearity Assumption

Component-plus-residual (partial residual) plots

The linearity assumption can be checked by examining plots of E_j against each X_j variable, but as pointed out in the text, these plots cannot distinguish between monotone and nonmonotone nonlinearity. An alternative is to use a component-plus-residual (otherwise called partial residual) plot. The partial residual for observation i for the variable X_j is

$$E_i^{(j)} = E_i + B_j X_{ij},$$

and we plot $E_i^{(j)}$ against X_{ij} to investigate nonlinearity. As shown in the text, we can also plot the partial residuals against the partial fits, which are predicted values obtained from the part of the fitted model using only terms involving X_{ij}. These ideas can be generalized for models with interaction, for example, by creating separate plots for subgroups. Cook (1993) has shown that partial residual plots will correctly model the relationship between the partial residuals and X_j if either i) X_j is linearly related to Y, or ii) the other covariates in the model are linearly related to X_j. The second condition suggests that by transforming covariates to linearize their interrelationships, we can improve the effectiveness of the partial residual plot. Generalizations of these plots have been suggested by Mallows (1986) and Cook (1993).

Discrete-valued covariates

Having discrete-valued covariates provides opportunities for testing linearity and homogeneity of variance, as illustrated in the text and the SAS examples.