Math-310 Homework 4 Due on Friday, 9/29/2017

This assignment requires Matlab. Please look at a sample solution in the next page.

1. Solve the initial value problem $\frac{dy}{dx} = y \sin x$, $y(0) = 1$, $0 \le x \le 4\pi$. Then use Euler's method with $N = 50$ (i.e. $h = 4\pi/50$) to solve it numerically. Compare the exact value $y(4\pi)$ with your approximated value (which should be the last value of your sequence $y_1, y_2, ...$). Then, plot the graph of the approximated solution and the graph of the exact solution in one figure.

2. Repeat Problem 1 with the Improved Euler method.

3. Repeat Problem 1 with the Runge-Kutta method.

4. Use Euler's method to solve $\frac{dy}{dx} = y \cos x + x$, $y(0) = 1$, $0 \le x \le 4\pi$ with $N = 50, 500, 5000$. Compare the three approximated values for $y(4\pi)$, and plot the graph of the three approximated solutions in one figure.

5. Solve the Problem 4 using Euler's method with $N = 5000$. (Yes, you did it in Problem 4.) Then, use the Improved Euler method with $N = 50$ to solve the problem. Compare the two approximated values for $y(4\pi)$, and plot the graph of the two approximated solutions in one figure.

6. Repeat Problem 5 with Runge-Kutta method instead of the Improved Euler method.

7. Solve $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$, $0 \le x \le 0.5$ using the Runge-Kutta method with $N = 50, 200, 5000$. Compare the three approximated values for $y(0.5)$.

8. Repeat Problem 7 for $0 \le x \le 0.97$. If you see anything unexpected, please read Example 5 on page 111.

Example. Solve the initial value problem $\dfrac{dy}{dx} = y\cos x,\ y(0) = 1,\ 0 \le x \le 4\pi$. Then use Euler's method with $N = 200$ (i.e. $h = 4\pi/200$) to solve it numerically. Compare the exact value $y(4\pi)$ with your approximated value (which should be the last value of your sequence $y_1, y_2, \ldots$. Then, plot the graph of the approximated solution and the graph of the exact solution in one figure.

Here is the code I use to do the problem

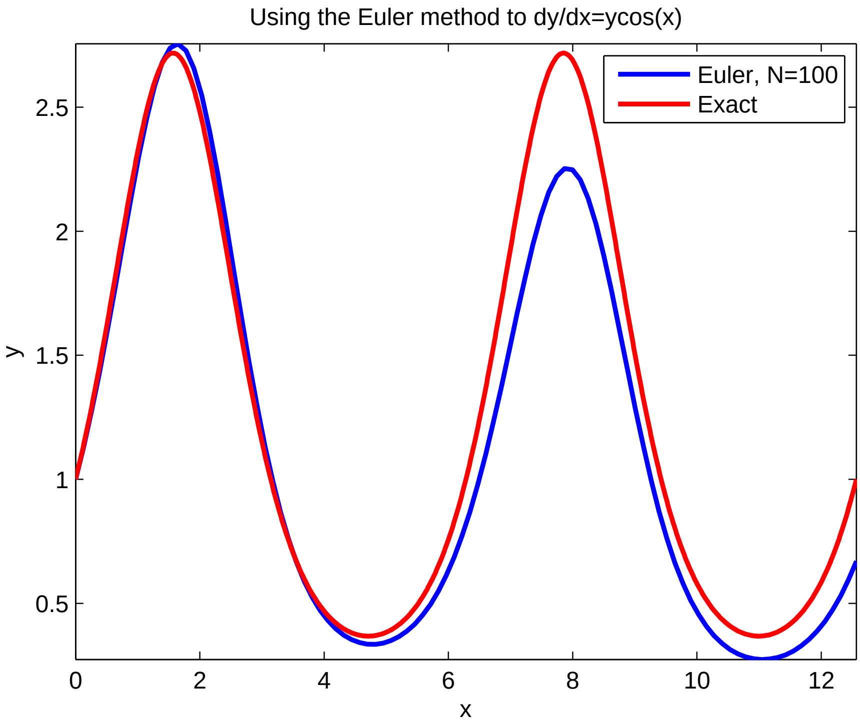
```
clc; clf; clear all
f=@(x,y) y*cos(x);
x_0=0;
y_0=1;
b=4*pi;

% Euler's method
N=100;
[x,y]=Euler(f,x_0,y_0,b,N);
plot(x,y,'b','LineWidth',2,'DisplayName','Euler, N=100');
hold on

% Exact solution
x=linspace(0,4*pi,1000);
z=exp(sin(x));
plot(x,z,'r','LineWidth',2,'DisplayName','Exact');

title('Using the Euler method to dy/dx=ycos(x)')
xlabel('x')
ylabel('y')
axis tight
legend('show')
hold off

fprintf('Problem 1:  dy/dx=ycos(x), y(0)=1. Euler \n')
fprintf('Approximated value = %0.6f\n',y(N))
fprintf('Exact value = %0.6f\n',z(end))
```

In this code, I called a function "Euler" which is defined below, and saved as in a separate file "Euler.m".

```
function [x,y]=Euler(f,x_0,y_0,b,N);
    h=(b-x_0)/(N-1);
    x=linspace(x_0,b,N);
    y(1)=y_0;
    for n=1:N-1
        y(n+1)=y(n)+h*f(x(n),y(n));
    end;
end
```

What you need to turn in is the output and the graph:

```
Problem 1:  dy/dx=ycos(x), y(0)=1. Euler
Approximated value = 0.669514
Exact value = 1.000000
```



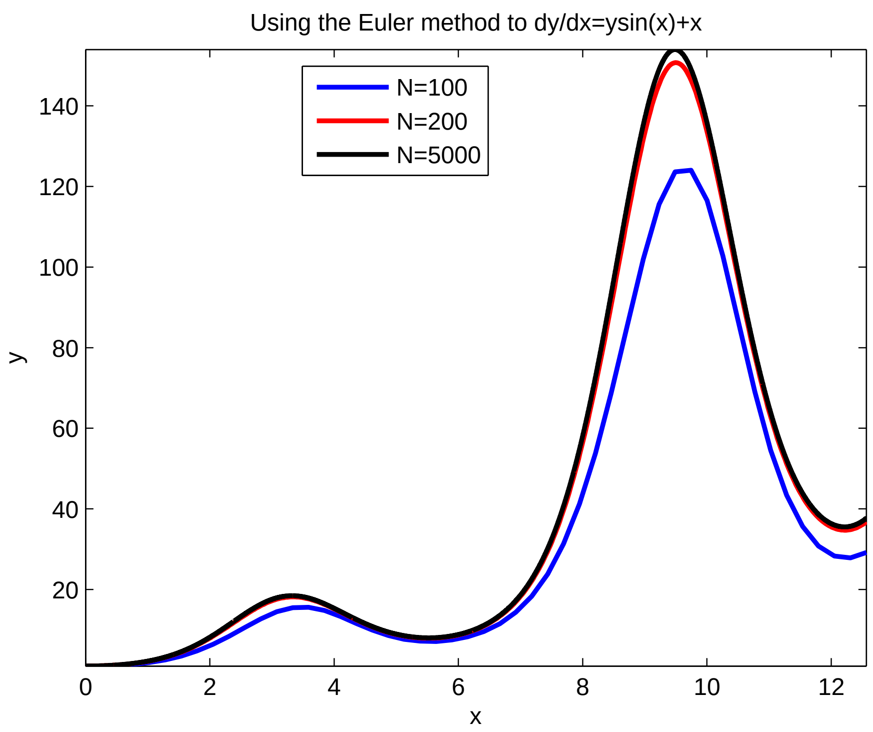
Example 2. Use Euler's method to solve $\frac{dy}{dx} = y\cos x + x$, $y(0) = 1$, $0 \leq x \leq 4\pi$ with $N = 50, 500, 5000$. Compare the three approximated values for $y(4\pi)$, and plot the graph of the three approximated solutions in one figure.
Here is the code:

```
clc; clear all; clf

f=@(x,y) y*sin(x)+x;
x_0=0;
y_0=1;
b=4*pi;

N=50;
[x,y]=Euler(f,x_0,y_0,b,N);
fprintf('Problem 4:  dy/dx=ysin(x)+x, y(0)=1. Euler \n')
fprintf('   N                 y(4pi)\n')
fprintf('%5d            %0.6f\n', N,y(N));
plot(x,y,'b','LineWidth',2,'DisplayName','N=100');
hold on

N=500;
[x,y]=Euler(f,x_0,y_0,b,N);
fprintf('%5d            %0.6f\n', N,y(N));
```

```
plot(x,y,'r','LineWidth',2,'DisplayName','N=200');

N=5000;
[x,y]=Euler(f,x_0,y_0,b,N);
fprintf('%5d             %0.6f\n', N,y(N));
plot(x,y,'k','LineWidth',2,'DisplayName','N=5000');

title('Using the Euler method to dy/dx=ysin(x)+x')
xlabel('x')
ylabel('y')
axis tight
legend('show')
hold off
```

Things to turn in: the output and the graph:

```
Problem 4:  dy/dx=ysin(x)+x, y(0)=1. Euler
   N               y(4pi)
   50            29.181331
  500            36.765816
 5000            37.672109
```



Note that for Euler's method, you may use the function *Euler* I defined. However, in order to to problems for Improved Euler or Runge-Kutta, you need to define the functions by yourself, say *ImprovedEuler* and *RungeKutta*, and call these functions when needed.