37.  \( h(x) = xf(x) \)

\[
h'(x) = \frac{d}{dx} [xf(x)] \\
= xf''(x) + f(x)
\]

38.  \( h(x) = (x^2 + 2x - 1)f(x) \)

\[
h'(x) = \frac{d}{dx} [(x^2 + 2x - 1)f(x)] \\
= (x^2 + 2x - 1)f'(x) + (2x + 2)f(x)
\]

61.  \( f(x) = \frac{1}{x}, \ g(x) = x^3 \)

a.  Using the product rule,

\[
\frac{d}{dx} \left[ \left( \frac{1}{x} \right) x^3 \right] = \left( \frac{1}{x} \right) (3x^2) + x^3 (-1)x^{-2} \\
= 3x - x = 2x = \frac{d}{dx} [x^2]
\]

b.  \( f'(x) = -\frac{1}{x^2} \) and \( g'(x) = 3x^2 \), so

\[
f'(x)g'(x) = -3.
\]

Now \( f(x)g(x) = \left( \frac{1}{x} \right) x^3 = x^2 \) which has derivative \( 2x \).

\( f'(x)g'(x) \neq (f(x)g(x))' \).
23. a. \( f''(9) < 0 \), so \( f(x) \) is decreasing at \( x = 9 \).

b. The function \( f(x) \) is increasing for \( 1 \leq x < 2 \) because the values of \( f''(x) \) are positive. The function \( f(x) \) is decreasing for \( 2 < x \leq 3 \) because the values of \( f''(x) \) are negative. Therefore, \( f(x) \) has a relative maximum at \( x = 2 \). Since \( f(2) = 9 \), the coordinates of the relative maximum point are \((2, 9)\).

c. The function \( f(x) \) is decreasing for \( 9 \leq x < 10 \) because the values of \( f''(x) \) are negative. The function \( f(x) \) is increasing for \( 10 < x \leq 11 \) because the values of \( f''(x) \) are positive. Therefore, \( f(x) \) has a relative minimum at \( x = 10 \).

d. \( f''(2) < 0 \), so the graph is concave down.

e. \( f'''(x) = 0 \), so the inflection point is at \( x = 6 \). Since \( f(6) = 5 \), the coordinates of the inflection point are \((6, 5)\).

f. The \( x \)-coordinate where \( f'(x) = 6 \) is \( x = 15 \).
24. a. \( f(2) = 3 \)

b. \( t = 4 \) or \( t = 6 \)

c. \( f(t) \) attains its greatest value after 1 minute, \( a = 1 \). To confirm this, observe that \( f'(t) > 0 \) for \( 0 \leq t < 1 \) and \( f'(t) < 0 \) for \( 1 < t \leq 2 \).

d. \( f(t) \) attains its least value after 5 minutes, at \( t = 5 \). To confirm this, observe that \( f'(t) < 0 \) for \( 4 \leq t < 5 \) and \( f'(t) > 0 \) for \( 5 < t \leq 6 \).

e. Since \( f'(7.5) = 1 \), the rate of change is 1 unit per minute.

f. The solutions to \( f'(t) = -1 \) are \( t = 2.5 \) and \( t = 3.5 \), so \( f'(t) \) is decreasing at the rate of 1 unit per minute after 2.5 minutes and after 3.5 minutes.

g. The greatest rate of decrease occurs when \( f'(t) \) is most negative, at \( t = 3 \) (after 3 minutes).

h. The greatest rate of increase occurs when \( f'(t) \) is most positive, at \( t = 7 \) (after 7 minutes).