Review 1

1. Review the definition of Poisson Processes (Definition 2.1.1), and try the following problem:
   Let \( N(t) \) be a Poisson process with mean \( \lambda \). Prove or disprove that
   (i) \( N_1(t) := N(t + 1) - N(1) \) is a Poisson process with rate \( \lambda \);
   (ii) \( N^*(t) := N(2t) - N(t) \) is a Poisson process with mean \( \lambda \).

2. Review Proposition 2.2.1, and try the following problem:
   Suppose cars pass Third street according to a Poisson process with a rate of 10 cars per minute. Frank wants to cross the street. He waits until there is no car come in the next 5 seconds. What is his expected waiting time?

3. Review the argument of conditioning on \( N(t) \) (page 66), and try the following problem:
   Let \( S_n \) be the time of \( n \)th event of the Poisson process \( N(t) \). Find \( \mathbb{E}[S_i|N(t) = n] \) (Hint: consider two different cases separately: \( i \leq n \) and \( i > n \).)

4. Review formula (2.4.1) for non-homogeneous Poisson processes, and try this problem:
   Bus arrives the stop at the intersection of Line street and Sixth street according to a non-homogeneous Poisson process with intensity function \( \lambda(t) \). Frank decides to wait for a bus for up to time \( s \). Find Frank’s expected waiting time.

5. Review the entire section of 2.5, and try the following problem:
   Suppose that cars pass a certain checkpoint on a highway according to a Poisson process with rate 60 cars per minute. The number of passengers in the cars are independent with the common distribution:
   \( \mathbb{P}(Y = 1) = 0.5, \mathbb{P}(Y = 2) = 0.3, \mathbb{P}(Y = 3) = 0.15 \) and \( \mathbb{P}(Y = 4) = 0.05 \). A car is full if it has four passengers.
   (i) Find the expected number of passengers that pass in the next minute.
   (ii) Find the probability that two full cars pass in the next 10 seconds.

6. Review the definition of renewal processes, formula (3.2.2) and Proposition 3.2.1, and try the following problem:
   Consider a renewal process \( N(t) \) whose interarrival times \( X_1, X_2, \ldots \) are i.i.d with pdf \( f(x) \). Denote \( m(t) = \mathbb{E}[N(t)] \) and \( v(t) = \mathbb{E}[N(t)^2] \). By conditioning on \( X_1 \), prove that
   \[
   v(t) = \int_0^t [1 + 2m(t - x) + v(t - x)] f(x) dx.
   \]