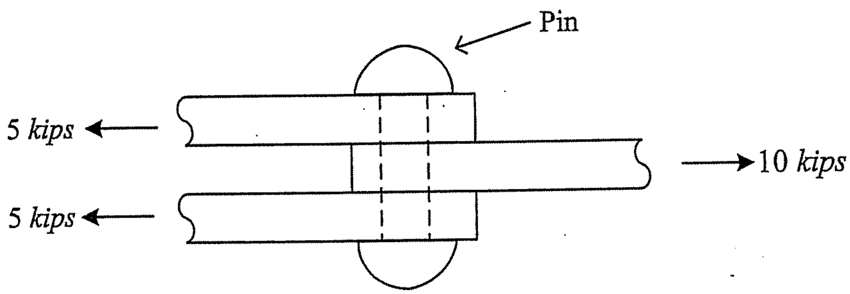
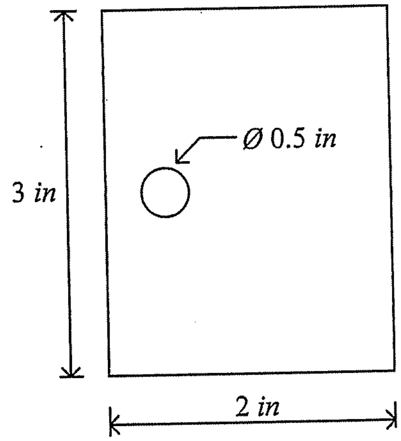
ENGR 350 - Mechanics of materials
Exam 1 - Sept 29, 2014

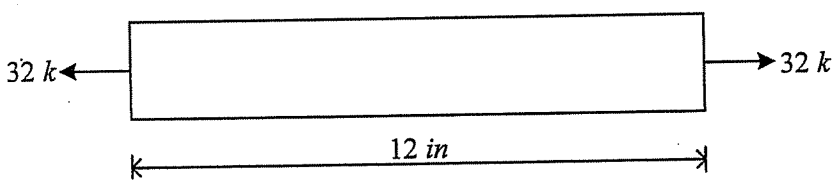
1) A pin is used to transmit the force from the middle plate to the two plates at the top and bottom as shown. The thickness of each plate is 0.5 in. Determine the following:

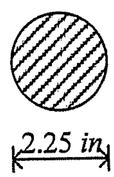
a) stress in the plate at a section away from the hole;
b) stress in the plate at the hole;
c) shear stress in the bolt;
d) bearing stress on the top plate;
e) bearing stress on the middle plate.



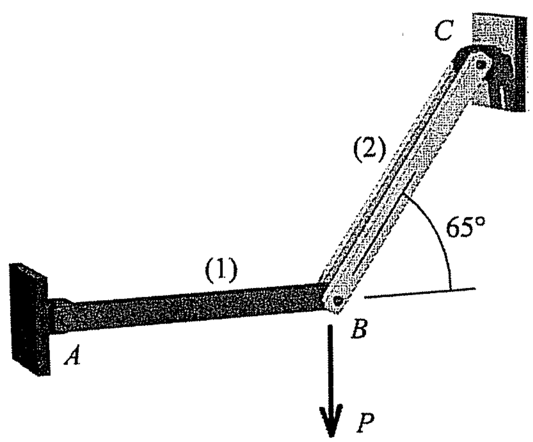


2) A circular rod made of aluminum is 2.25 in. in diameter and 12 in. in length. The rod is subjected to an axial force of 32 kips. At the end of the tension test, the rod length was measured to be 12.00938 in. and its diameter decreased to 2.249415 in. Determine the two elastic constants for aluminum, namely Young's modulus E and Poisson's ratio ν.





3) Member (1) is a steel bar with a cross-sectional area of 1.35 in.^{2} and a yield strength of 50 ksi. Member (2) is a pair of 6061-T6 aluminum bars having a combined cross-sectional area of 3.50 in.^{2} and a yield strength of 40 ksi. A factor of safety of 1.6 with respect to yield is required for both members. Determine the maximum allowable load P that may be applied to the structure.



① (a)

$$\sigma = \frac{F}{A} = \frac{10\ \text{Kips}}{3 \cdot 0.5\ \text{in}^2} = 6.667\ \text{ksi} = 6667\ \text{psi}$$

(b) $$\sigma = \frac{F}{A - t \cdot d_{bolt}} = \frac{10\ \text{kips}}{3 \cdot 0.5 - 0.5 \cdot 0.5} = 8\ \text{ksi} = 8000\ \text{psi}$$

(c) $$\tau = \frac{F_{shear}}{A_{shear}} = \frac{5\ \text{Kips}}{\frac{\pi \cdot (0.5\ \text{in})^2}{4}} = 25.46\ \text{ksi} = 25{,}460\ \text{psi}$$

(d) $$\sigma_{b_{top}} = \frac{F_{bearing}}{A_{bearing}} = \frac{5\ \text{kips}}{0.5 \cdot 0.5\ \text{in}^2} = 20\ \text{ksi} = 20{,}000\ \text{psi}$$

(e) $$\sigma_{b_{middle}} = \frac{10\ \text{Kips}}{0.5 \cdot 0.5\ \text{in}^2} = 40\ \text{ksi} = 40{,}000\ \text{psi}$$

(2)

$L_0 = 12$ in
$L = 12.00938$ in

$\Rightarrow \delta = L - L_0 = 0.00938$ in

$\varepsilon = \frac{\delta}{L_0} = \frac{0.00938}{12} = 7.81 \cdot 10^{-4}$

$A = \frac{\pi d^2}{4} = \frac{\pi \cdot 2.25^2}{4} = 3.976 \text{ in}^2$

$\sigma = \frac{F}{A} = \frac{32 \cdot 10^3 \text{ lb}}{3.976 \text{ in}^2} = 8048 \text{ psi}$

$\sigma = E\varepsilon \Rightarrow E = \frac{\sigma}{\varepsilon} = \frac{8048 \text{ psi}}{7.81 \cdot 10^{-4}} = 1030.47 \cdot 10^4 \text{ psi}$

$= 10.3 \cdot 10^6 \text{ psi} = 10{,}300 \text{ ksi} = 10.3 \text{ Mpsi}$

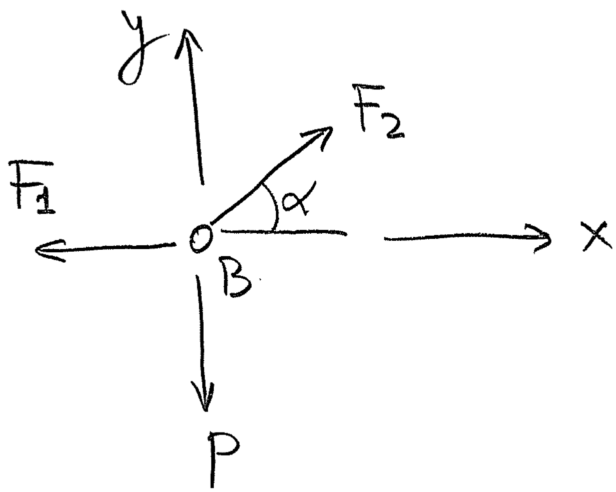
$\varepsilon_{long} = 7.81 \cdot 10^{-4}$

$\varepsilon_{transv.} = \frac{d - d_0}{d_0} = \frac{2.249415 - 2.25}{2.25} = -2.6 \cdot 10^{-4}$

$\nu = -\frac{\varepsilon_{transv.}}{\varepsilon_{long}} = \frac{2.6 \cdot 10^{-4}}{7.81 \cdot 10^{-4}} = 0.33$

(3) Equilibrium at B :

$\alpha = 65°$



$\sum F_x = 0 \iff F_1 = F_2 \cos\alpha$

$\sum F_y = 0 \iff F_2 \sin\alpha = P \Rightarrow F_2 = P \frac{1}{\sin\alpha}$ } $\Rightarrow$

$\Rightarrow F_1 = P \frac{\cos\alpha}{\sin\alpha}$

Stress in bar (1):

$\sigma_1 = \frac{F_1}{A_1}$

Stress in each bar (2):

$\sigma_2 = \frac{F_2}{A_2}$

Factors of safety :

$FS_1 = \frac{\sigma_{y_1}}{\sigma_1} \iff FS = \frac{\sigma_{y_1}}{\frac{F_1}{A_1}} = \frac{\sigma_{y_1}}{P \frac{\cos\alpha}{A_1 \sin\alpha}} = \frac{\sigma_{y_1} A_1 \sin\alpha}{P\cos\alpha}$

$\Rightarrow P = \frac{\sigma_{y_1} A_1 \sin\alpha}{FS \cos\alpha} = \frac{50 \cdot 10^3 \cdot 1.35 \sin 65°}{1.6 \cdot \cos 65°} = 90.47 \cdot 10^3 \text{ lb}$

$FS = \frac{\sigma_{y_2}}{\sigma_2} \iff FS = \frac{\sigma_{y_2}}{\frac{F_2}{A_2}} = \frac{\sigma_{y_2}}{P \frac{1}{A_2 \sin\alpha}} = \frac{\sigma_{y_2} A_2 \sin\alpha}{P} \Rightarrow$

$$\Rightarrow P = \frac{\sigma_{y_2} A_2 \sin\alpha}{FS} = \frac{40 \cdot 10^3 \cdot 3.5 \cdot \sin 65^\circ}{1.6} = 79.30 \cdot 10^3 \text{ lb}$$

Choose

$$\boxed{P = 79.30 \cdot 10^3 \text{ lb} = 79.30 \text{ kips}}$$