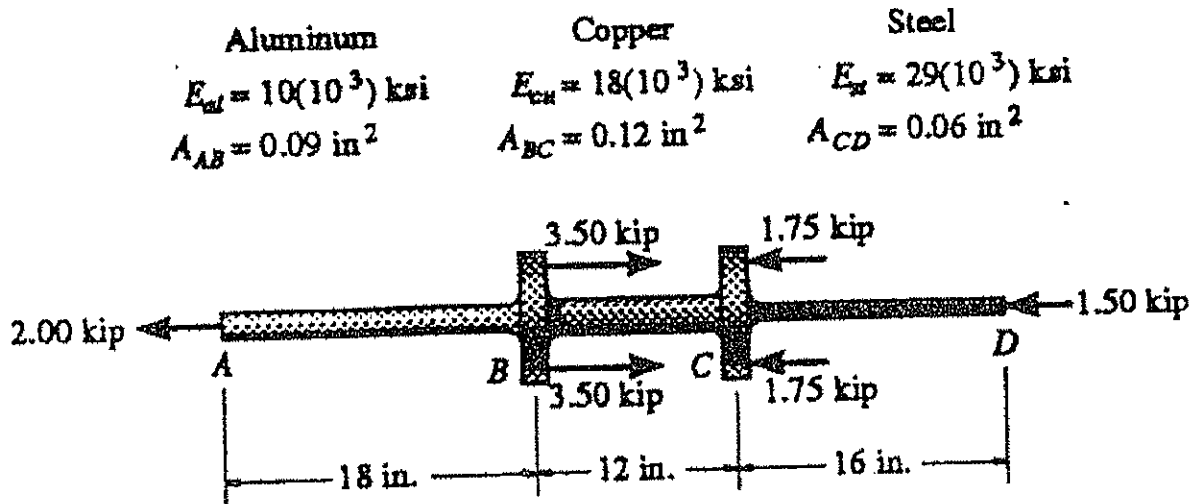


**ENGR 350 Mechanics of Materials**  
**Final exam, Monday, Dec. 15, 2014. 3:00-5:00 pm**

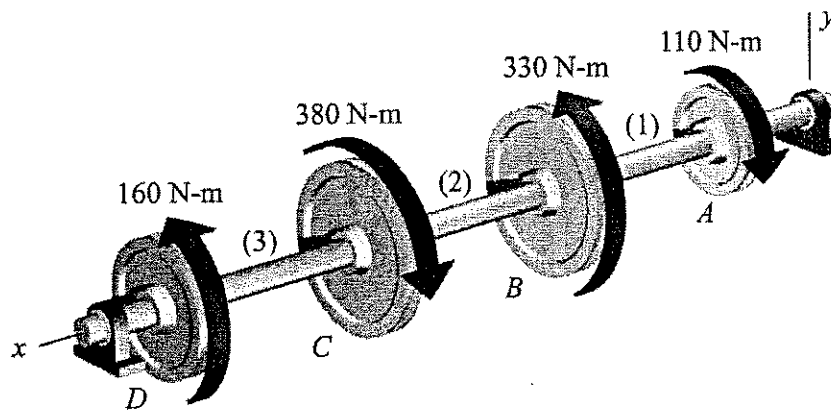
1. (25 pts.) The composite beam, consisting of aluminum, copper, and steel sections, is subjected to the loading shown. Determine the displacement of end A with respect to end D and the normal stress in each section. The cross-sectional area and modulus of elasticity for each section are shown in the figure. Neglect the size of the collars at B and C.



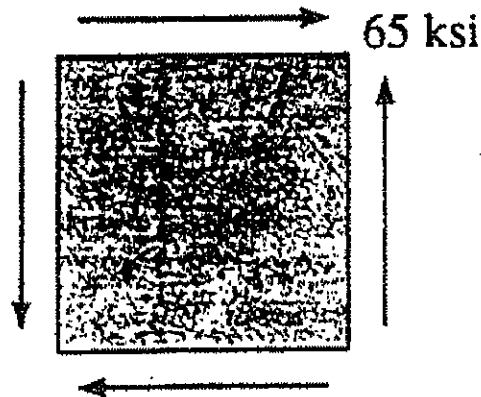
2) (25 pts.) A solid constant-diameter shaft is subjected to the torques shown in the figure. The bearings shown allow the shaft to turn freely.

(a) Plot a torque free-body diagram with the internal torque for each segment (1), (2), and (3) of the shaft.

(b) If the allowable shear stress in the shaft is 80 MPa, determine the minimum acceptable diameter for the shaft.

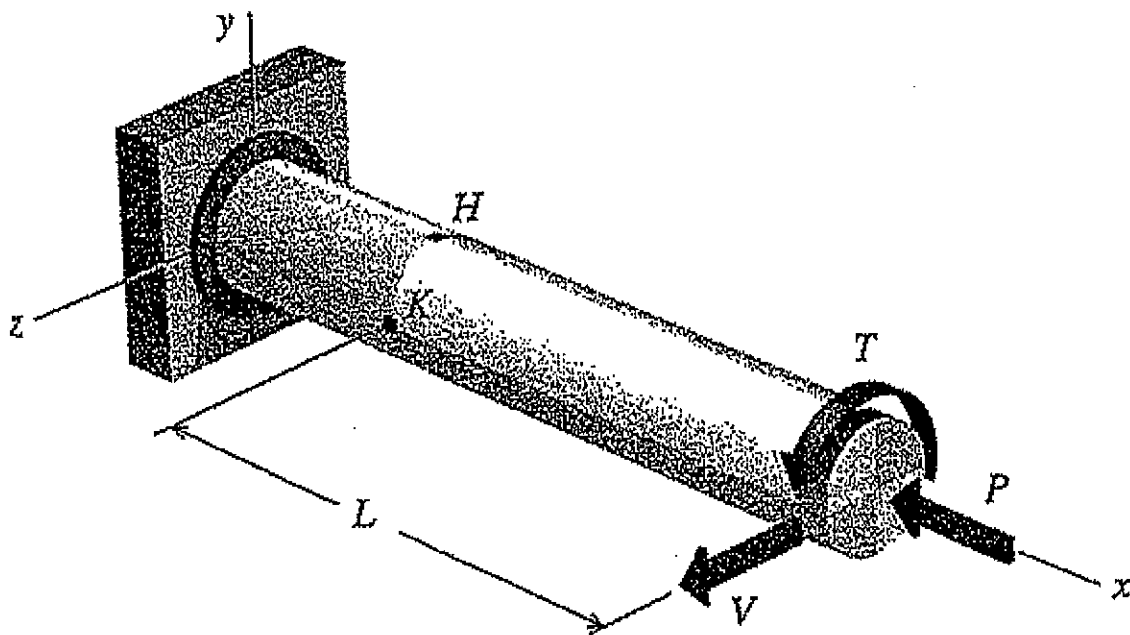


3. (25 pts.) Using Mohr's circle for the stress state shown below, determine:
- the stress state if the element shown is rotated  $60^\circ$  clockwise from the orientation shown;
  - principal in-plane stresses;
  - maximum in-plane shear stress.



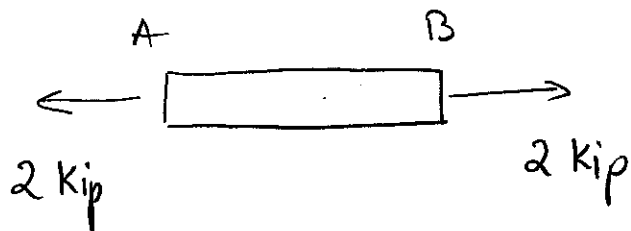
- 4) (25 pts.) A 1.25-in. -diameter solid shaft is subjected to an axial force of  $P = 520$  lb, a horizontal shear force of  $V = 275$  lb and a concentrated torque of  $T = 880$  lb-in. Assume  $L = 7.0$  in. Determine:

- The stresses at point H and sketch the stress components on a 3D cubic element.
- The stresses at point K and sketch the stress components on a 3D cubic element.
- Represent the stress state at point K on the Mohr's circle. (Build the Mohr's circle by computing the principal stresses, circle center and radius).

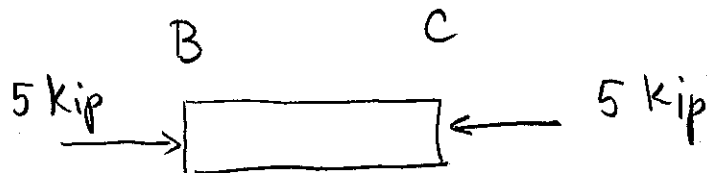


①

$$\tau = \frac{F}{A} \quad \delta = \frac{FL}{EA}$$



$$\tau_{AB} = \frac{2 \text{ Kip}}{0.09 \text{ in}^2} = 22.2 \text{ Ksi (T)}$$



$$\tau_{BC} = \frac{5 \text{ Kip}}{0.12 \text{ in}^2} = 41.7 \text{ Ksi (C)}$$



$$\tau_{CD} = \frac{1.5 \text{ Kip}}{0.06 \text{ in}^2} = 25.0 \text{ Ksi (C)}$$

$$\delta_{AB} = \frac{2 \cdot 18}{0.09 \cdot 10 \cdot 10^3} = 0.04$$

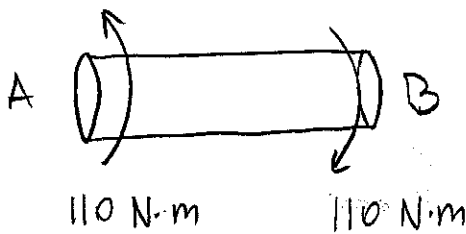
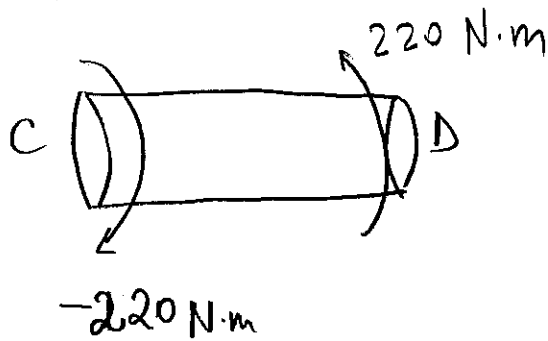
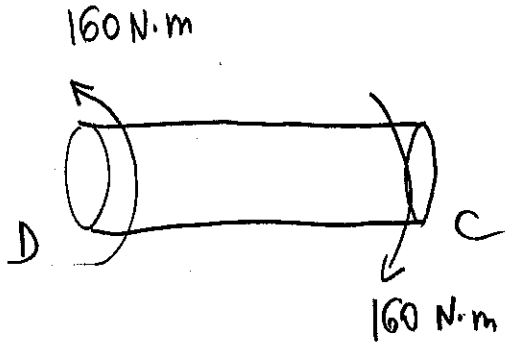
$$\delta_{BC} = -\frac{5 \cdot 12}{0.12 \cdot 18 \cdot 10^3} = -0.0277$$

$$\delta_{CD} = \frac{-1.5 \cdot 16}{0.06 \cdot 29 \cdot 10^3} = -0.01379$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = -0.00157$$

2

(a)



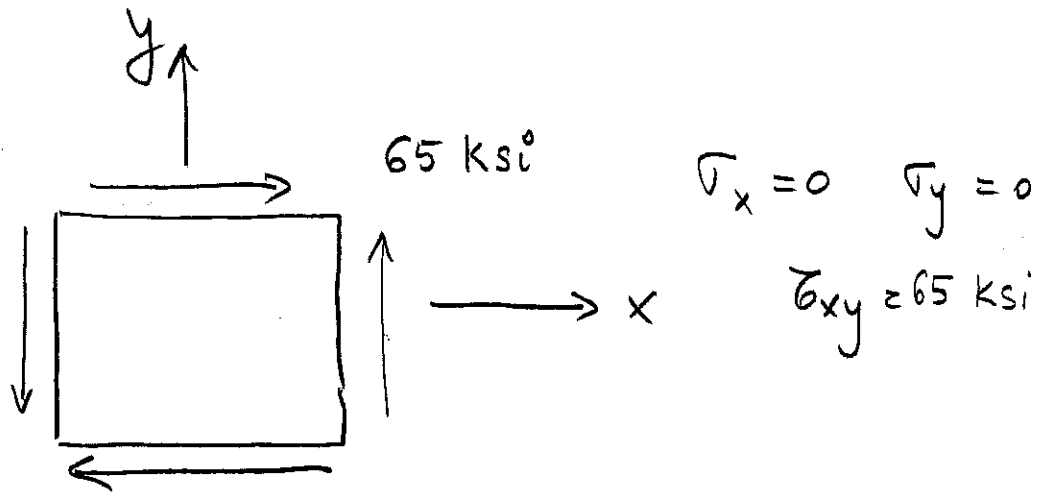
$$(b) \quad \tau = \frac{T r}{J} = \frac{T \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{16 T}{\pi d^3} = \tau_{all} \Rightarrow$$

$$\Rightarrow d = \sqrt[3]{\frac{16 T}{\pi \tau_{all}}}$$

Using the largest torque :

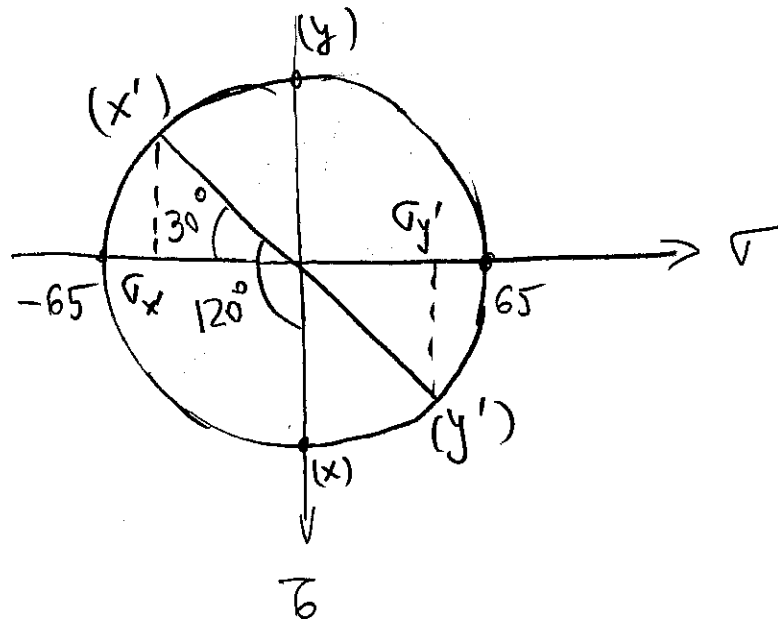
$$d = \sqrt[3]{\frac{16 \cdot 220 \cdot 10^3}{\pi \cdot 80}} = 24.1 \text{ mm}$$

③



$$(a) \quad \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm \tau_{xy} \begin{cases} \sigma_1 = 65 \text{ ksi} \\ \sigma_2 = -65 \text{ ksi} \end{cases}$$

$$\tau_{\max} = R = \frac{\sigma_1 - \sigma_2}{2} = 65 \text{ ksi}$$



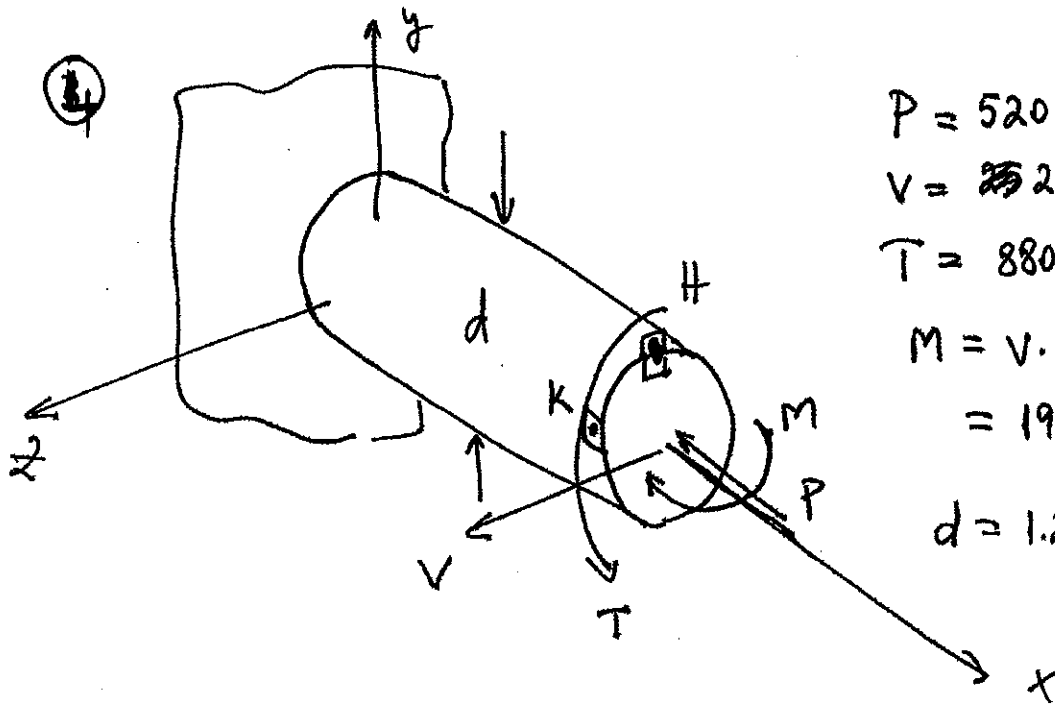
$$\sigma_{x'} = -R \cos 30^\circ = -65 \cos 30^\circ = -56.3 \text{ ksi}$$

$$\sigma_{y'} = R \cos 30^\circ = 65 \cos 30^\circ = 56.3 \text{ ksi}$$

$$\tau_{xy'} = -R \sin 30^\circ = -65 \sin 30^\circ = -32.5 \text{ ksi}$$

(b)  $\sigma_1 = 65 \text{ ksi}$       $\sigma_2 = -65 \text{ ksi}$

(c)  $\tau_{\max} = 65 \text{ ksi}$



$$\begin{aligned}
 P &= 520 \text{ lb} \\
 V &= 275 \text{ lb} \\
 T &= 880 \text{ lb}\cdot\text{in} \\
 M &= V \cdot L = 275 \cdot 7 \\
 &= 1925 \text{ lb}\cdot\text{in} \\
 d &= 1.25 \text{ in}
 \end{aligned}$$

(a) Stresses at H

$$\sigma_P = -\frac{P}{A} = -\frac{P}{\frac{\pi d^2}{4}} = -\frac{4P}{\pi d^2} = -\frac{4 \cdot 520}{\pi \cdot (1.25)^2} = -423.7 \text{ psi}$$

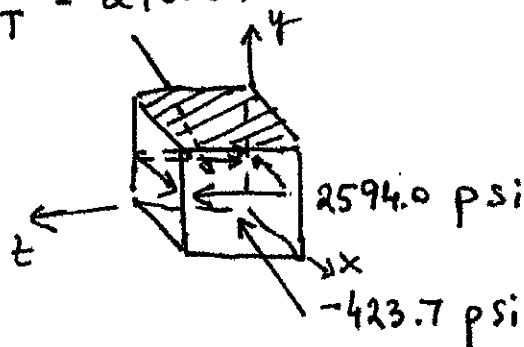
$$\sigma_M = 0$$

$$\sigma_V = \frac{4V}{3A} = \frac{4}{3} \frac{V}{\frac{\pi d^2}{4}} = \frac{16V}{3\pi d^2} = \frac{16 \cdot 275}{3 \cdot \pi \cdot (1.25)^2} = 298.8 \text{ psi}$$

$$\sigma_T = \frac{Tr}{J} = \frac{T \cdot \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{16T}{\pi d^3} = \frac{16 \cdot 880}{\pi \cdot (1.25)^3} = 2295 \text{ psi}$$

$$\sigma = \sigma_V + \sigma_T = 298.8 + 2295.0 = 2594.0 \text{ psi}$$

(H)



(b) Stresses at K :

$$\sigma_p = -423.7 \text{ psi}$$

$$\sigma_M = -\frac{My}{I} = -\frac{M \frac{d}{2}}{\frac{\pi d^4}{64}} = -\frac{32M}{\pi d^3} = -\frac{32 \cdot 1925}{\pi (1.25)^3} = -10,040 \text{ psi}$$

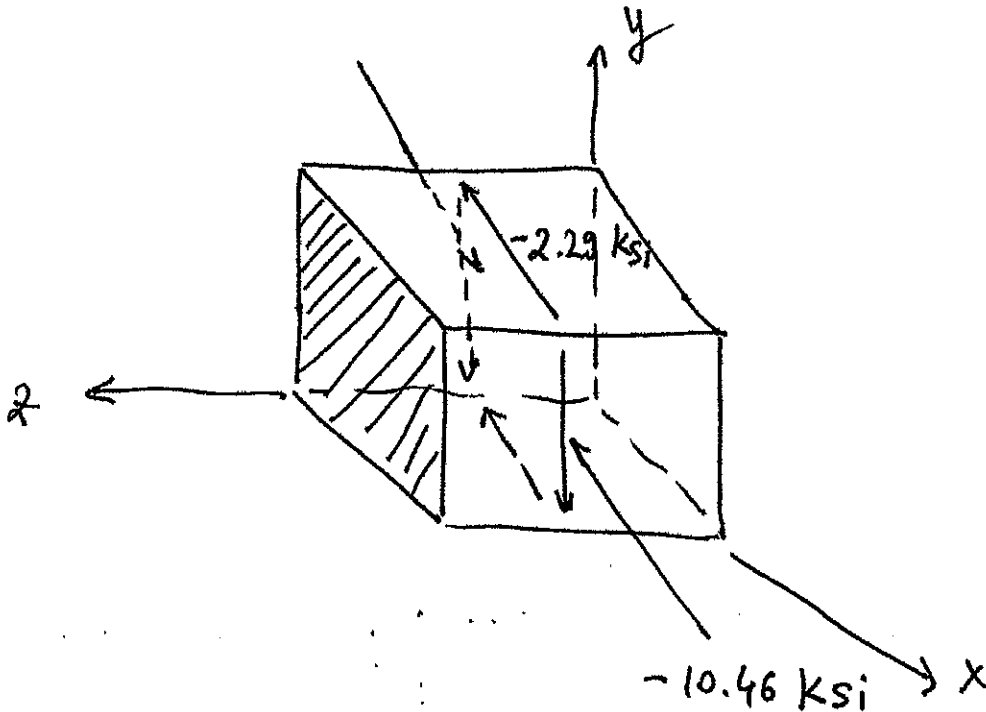
$$\tau_v = 0$$

$$\tau_T = -2295 \text{ psi}$$

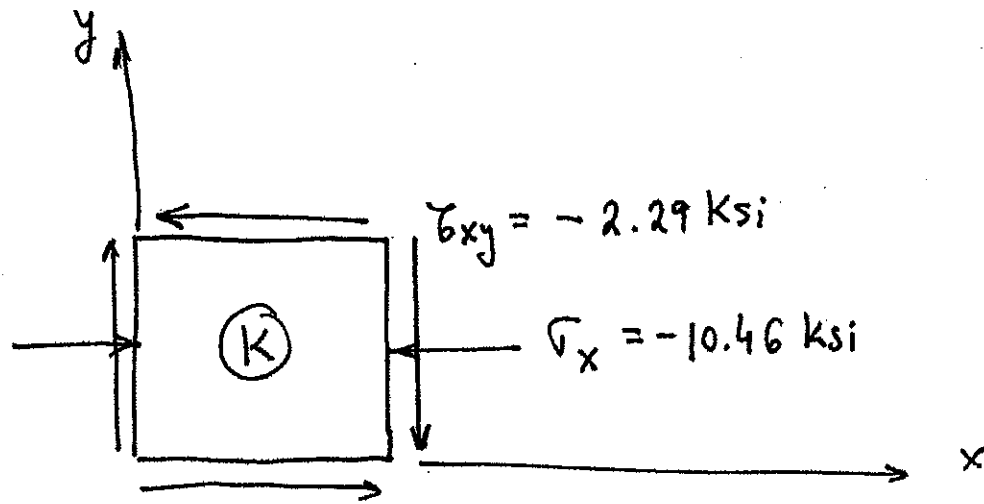
So,

$$\sigma = \sigma_p + \sigma_M = -423.7 - 10,040 = -10,463.7 \text{ psi}$$
$$= -10.46 \text{ ksi}$$

$$\tau = \tau_T = -2.29 \text{ ksi}$$



(c)



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = -\frac{10.46}{2} + \sqrt{\left(\frac{10.46}{2}\right)^2 + (2.29)^2} = 0.48 \text{ ksi}$$

$$\sigma_2 = -\frac{10.46}{2} - \sqrt{\left(\frac{10.46}{2}\right)^2 + (2.29)^2} = -10.94 \text{ ksi}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5.7 \text{ ksi}$$

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{-10.46}{2} = -5.23 \text{ ksi}$$