ENGR 350 - Mechanics of Materials, Fall 2013

Homework #1

Due: Fri., Sept. 5

1. Problem P1.5, p. 17
2. Problem P1.8, p. 17
3. Problem P1.16, p. 18
4. Problem P1.26, p. 20
5. Problem P1.27, p. 20
6. Problem P1.29, p. 21

(all problems are from the textbook).
1.5

Rod (1) \( d_1 = 2 \text{ in. Aluminum} \)
Rod (2) \( d_2 = 1.5 \text{ in. Brass} \)
Rod (3) \( d_3 = 3.0 \text{ in. Steel} \)

Determine normal stresses in each:

FBD's

- **Rod 1**
  \[ \frac{1}{2} F_1 = 0 \]
  
  \[ F_1 = (8 - 4 - 4) \text{kips} \]
  
  \[ F_1 = -11 \text{kips} \]

- **Rod 2**
  \[ F_2 = (15 + 15 - 4 - 4 - 8) \text{kips} \]
  
  \[ F_2 = 14 \text{kips} \]

- **Rod 3**
  \[ F_3 = (-20 - 20 + 15 + 15 - 8 - 4) \text{kips} \]
  
  \[ F_3 = (-21) \text{kips} \]

**Stresses**:

\[ \sigma_1 = \frac{-11 \text{kips}}{\frac{\pi}{4} (2 \text{ in.})^2} = \frac{-509 \text{ ksi}}{} \]

\[ \sigma_2 = \frac{14 \text{kips}}{\frac{\pi}{4} (1.5 \text{ m})^2} = \frac{792 \text{ ksi}}{} \]

\[ \sigma_3 = \frac{-21 \text{kips}}{\frac{\pi}{4} (3.0 \text{ m})^3} \]

\[ = -3.678 \text{ ksi} \]

\[ = 3.678 \text{ ksi} \]
\[ \tan \alpha = \frac{4.0 \text{m}}{5.2 \text{m}} = 0.768 \quad \alpha = 57.99^\circ \]
\[ \tan \beta = \frac{2.3 \text{m}}{3.2 \text{m}} = 0.7188 \quad \beta = 35.70^\circ \]

\[ 2F_x = F_2 \cos \beta = -F_1 \cos \alpha \]
\[ 2F_y = F_2 \sin \beta + F_1 \sin \alpha - P \]

Solve for \( F_1 + F_2 \):
\[ F_2 = F_1 \frac{\cos \alpha}{\cos \beta} \]

Plug into other eqn and solve for \( F_1 \):
\[ F_1 \frac{\cos \alpha}{\cos \beta} \sin \beta + F_1 \sin \alpha = P \]
\[ F_1 \left( \cos \alpha \tan \beta + \sin \alpha \right) = P \]
\[ F_1 = \frac{P}{1.2289} = \frac{50 \text{ KN}}{1.2289} = 40.6816 \text{ KN} \]

\[ F_2 = F_1 \frac{\cos \alpha}{\cos \beta} = 40.6816 \text{ KN} \frac{\cos 57.995^\circ}{\cos 35.70^\circ} = 216.553 \text{ KN} \]

\[ A_1 = \frac{F_1}{0.1} = \frac{(40.6816 \text{ KN}) \times (1000 \text{ N/KN})}{130 \text{ N/mm}^2} = 312.97 \text{ mm}^2 \]
312.97 mm\(^2 = \frac{\pi}{4} d^2 \quad d \geq 19.942 \text{ mm} \]

\[ A_2 = \frac{F_2}{0.2} = \frac{(216.553 \text{ KN}) \times (1000 \text{ N/KN})}{130 \text{ N/mm}^2} = 204.272 \text{ mm}^2 \]
204.272 mm\(^2 = \frac{\pi}{4} d^2 \quad d \geq 11.127 \text{ mm} \]
Problem 1.16

Given: Two 6 in wide wooden boards are to be joined by splice plates that will be fully glued on the contact surfaces. The glue to be used can safely provide a shear strength of 120 psi.

- gap of .5 in is required between boards 1 & 2

Find: Determine the smallest allowable length \( L \) that can be used for the splice plates for an applied load of \( P = 10,000 \text{ lb} \).

Solution: \( P = 10,000 \text{ lb} \) \( \sigma_s = 120 \text{ psi} \)

\[ \Sigma F_x = P - V - V = 0 \]

\[ P = 2V \]
\[ V = \frac{10,000 \text{ lb}}{2} \]
\[ V = 5000 \text{ lb} \]

\[ A = \frac{L_g (w)}{w} \]
\[ L_g = \frac{41.667 \text{ in}^2}{\sigma_s} \]
\[ L_g = 0.944 \text{ in} \]

\[ L = L_g \times \frac{w_1}{w} + g \]
\[ = 0.944 \text{ in} \times 6 \text{ in} + 6.944 \text{ in} + 0.5 \text{ in} \]

\[ L = 14.39 \text{ in} \]
Problem 1.26

Given: The rod diameter is 15 mm, the hole diameter is 20 mm, and the load is 225 MPa. The rod head is 12 mm wide and 10 mm thick.

Find: (a) bearing stress acting between the support plate and the rod head.
(b) the average shear stress produced in the rod head.
(c) the punching shear stress produced in the support plate by the rod head.

Solution:

(a) $A_{rod} = \pi/4 (15\text{mm})^2 = 176.715 \text{mm}^2$

$F_{rod} = F_{rod} A_{rod} = 225 \text{MPa} \times 176.715 \text{mm}^2 = 39760.782 \text{N}$

$A_{contact} = \pi/4 [(30\text{mm})^2 - (20\text{mm})^2] = 392.699 \text{mm}^2$

$V_b = \frac{F_{rod}}{A_{contact}} = \frac{39760.782 \text{N}}{392.699 \text{mm}^2} = 101.3 \text{MPa}$

(b) $A_u = \pi (15\text{mm}) (10\text{mm}) = 471.239 \text{mm}^2$

$z = \frac{39760.782 \text{N}}{471.239 \text{mm}^2} = 84.4 \text{MPa}$
\[ A_v = \pi (30 \text{ mm}) (12 \text{ mm}) \]
\[ = 1130.973 \text{ mm}^2 \]
\[ \sigma = \frac{39760.782 \text{ N}}{1130.973 \text{ mm}^2} \]
\[ \sigma = 35.2 \text{ MPa} \]
Problem 1.27

Given: The rectangular bar is connected to the support bracket with a circular pin. The bar width is \( w = 1.75 \text{ in.} \) and the bar thickness is \( t = .375 \text{ in.} \)

Find: For an applied load of \( P = 5600 \text{ lb} \) determine the average bearing stress produced in the bar by the .625 in diameter pin.

Solution: \( d_p = .625 \text{ in.} \) \( t = .375 \text{ in.} \) \( P = 5600 \text{ lb} \) \( w = 1.75 \text{ in.} \)

\[
\sigma_b = \frac{P}{d_p t} = \frac{5600 \text{ lb}}{(.625 \text{ in.})(.375 \text{ in.})} \]

\[
\sigma_b = 23893.33 \text{ psi}
\]
\[ D_{\text{pipe}} = 8\text{ in} \quad \text{Weight} = 2000\text{ lb} \]
\[ D_{\text{rod}} = 0.5\text{ in} \quad (\text{hanger rod}) \]
\[ D_{\text{bolt}} = 0.625\text{ in} \]

Bottom Strap: \( t = \frac{3}{16}\text{ in} \quad \omega = 1.75\text{ in} \quad l = 3l_0\text{ in} \)

**a)** normal stress in hanger rod

\[ A_{\text{rod}} = \frac{\pi}{4} (0.5\text{ in})^2 = 0.1964\text{ in}^2 \]
\[ \tau_{\text{rod}} = \frac{2000\text{ lb}}{0.1964\text{ in}^2} = 10,175.92\text{ psi} \]

**b)** shear stress in bolt

\[ A_{\text{bolt}} = \frac{\pi}{4} (0.625\text{ in})^2 = 0.3048\text{ in}^2 \]
\[ \tau_{\text{bolt}} = \frac{2000\text{ lb}}{2(0.3048\text{ in}^2)} = 3,259.50\text{ psi} \]

**c)**

\[ \tau_B = \frac{2000\text{ lb}}{2(0.625\text{ in})(\frac{3}{16}\text{ in})} = 8533.3\text{ psi} \]