ENGR 350 - Mechanics of Materials, Fall 2013

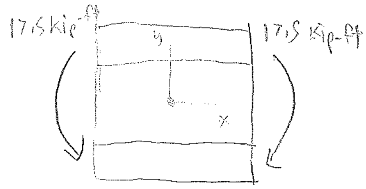
Homework #10

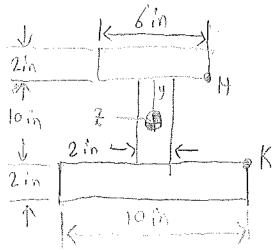
Due: Monday, Nov. 10

1. Problem P8.8, p. 277
2. Problem P8.10, p. 278
3. Problem P8.19, p. 289
4. Problem P8.52, p. 318
5. Problem P8.54, p. 318

(all problems are from the textbook).

8.8) ⟨Given⟩





⟨Goal⟩

(a) Centroid location, I_z, S_z

(b) Bending stress at point H

(c) Bending stress at point K

(d) Max bending stress in the cross-section

⟨Solution⟩

(a)

shape	A_i (in^2)	y_i (in)	$y_i A_i$ (in^3)
top flange	12	13	156
middle	20	7	140
bottom flange	20	1	20
	52		316

$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{316\ in^3}{52\ in^2} = \boxed{6.08\ in}$$

Shape	I_c (in^4)	$d = y_i - \bar{y}$ (in)	$d^2 A$ (in^4)	$I_c + d^2 A$ (in^4)
top flange	4.00	6.92	575.15	579.15
middle	166.67	0.92	17.04	183.71
bottom flange	6.67	-5.08	515.50	522.17
				1285.03

$\Rightarrow \boxed{I_z = 1285\ in^4}$

$C_{bot} = 6.08\ in$

$C_{top} = 14 - 6.08 = 7.92\ in$

$$S_{bot} = \frac{I_z}{C_{bot}} = \frac{1285\ in^4}{6.08\ in} = 211.5\ in^3$$

$$S_{top} = \frac{I_z}{C_{top}} = \frac{1285\ in^4}{7.92\ in} = 162.2\ in^3$$

$$\boxed{S = 162.2\ in^3}$$

8.8 cont.) Point H:

$$\sigma_x = -\frac{My}{I_z} = -\frac{(-17.5\text{ kip-ft})(7.92\text{ in} - 2\text{ in})(12\text{ in/ft})}{1285\text{ in}^4} = \boxed{968\text{ psi (T)}}$$

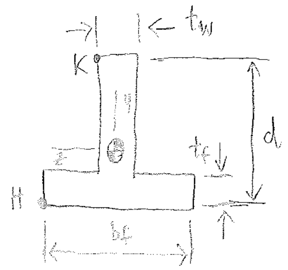
Point K:

$$\sigma_x = -\frac{My}{I_z} = -\frac{(-17.5\text{ kip-ft})(-6.08\text{ in} + 2\text{ in})(12\text{ in/ft})}{1285\text{ in}^4} = \boxed{666\text{ psi (C)}}$$

Max bending stress: $c_{top} > c_{bot} \Rightarrow \sigma_{max}$ at the top

$$\sigma_x = -\frac{My}{I_z} = -\frac{(-17.5\text{ kip-ft})(7.92\text{ in})(12\text{ in/ft})}{1285\text{ in}^4} = \boxed{1{,}295\text{ psi (T)}}$$

8.10) <Given>



d = 5.0 in

b_f = 4.0 in

t_f = 0.5 in

t_w = 0.25 in

σ_H = 4500 psi (T)

<Goal>

(a) Find M_z

(b) Find σ_x at point K

<Solution>

Shape	A_i (in^2)	y_i (in)	y_iA_i (in^3)
bottom flange	2.00	0.25	0.50
stem	1.13	2.75	3.09
	3.13		3.59

$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{3.59\ in^3}{3.13\ in^2} = \boxed{1.15\ in}$$

Shape	I_c (in^4)	$d = y_i - \bar{y}$ (in)	d^2A (in^4)	$I_c + d^2A$ (in^4)
bottom flange	0.042	-0.900	1.620	1.662
stem	1.898	1.600	2.880	4.778
				6.440

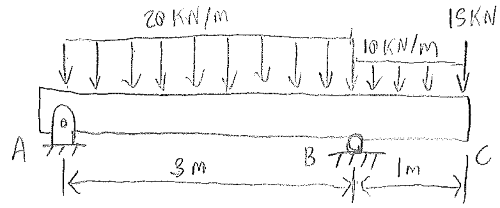
$\Rightarrow M_z = 6.44\ in^4$

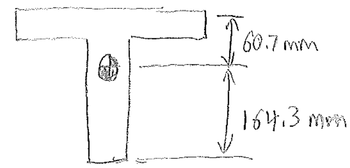
(a) $\sigma_x = -\frac{M_z y}{I_z}$

$$M_z = \frac{-\sigma_x I_z}{y} = -\frac{(4500\ psi)(6.44\ in^4)}{-1.15\ in} = 25{,}200\ lb\text{-}in = \boxed{2{,}100\ lb\text{-}ft}$$

(b) $$\sigma_x = \frac{-M_z y}{I_z} = -\frac{(25{,}200\ lb\text{-}in)(3.85\ in)}{6.44\ in^4} = -15{,}065\ psi = \boxed{15{,}070\ psi\ (C)}$$

8.19) <Given> WT230 x 26 steel





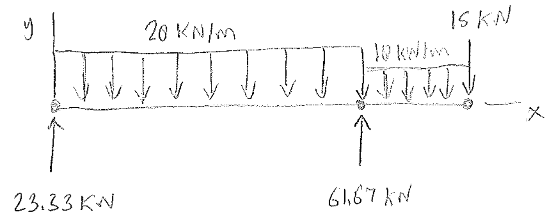
$I_z = 16.7 \times 10^6 \ mm^4$

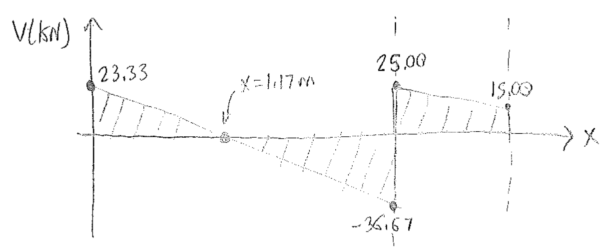
<Goal>

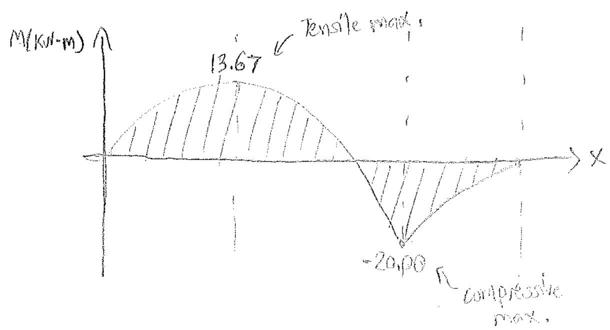
(a) Max. tensile bending stress

(b) Max. compressive bending stress

<Solution>







Positive max. stresses:

$$\sigma_x = \frac{-(13.61 \ KN\text{-}m)(60.7mm)(1000)^2}{16.7 \times 10^6 \ mm^4} = 49.5 \ MPa \ (C)$$

$$\sigma_x = \frac{-(13.61 \ KN\text{-}m)(-164.3mm)(1000)^2}{16.7 \times 10^6 \ mm^4} = 133.9 \ MPa \ (T) \ ✓$$

Negative max. stresses:

$$\sigma_x = -\frac{(-20 KN\text{-}m)(60.7mm)(1000)^2}{16.7 \times 10^6 \ mm^4} = 72.7 \ MPa \ (T)$$

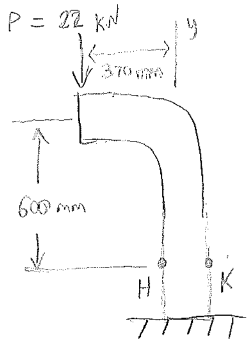
$$\sigma_x = \frac{-(-20 KN\text{-}m)(-164.3 mm)(1000)^2}{16.7 \times 10^6 mm^4} = 196.8 \ MPa \ (C) \ ✓$$

Overall Max. stresses:

(a) $\sigma_{max,T}$ = **133.9 MPa (T)**

(b) $\sigma_{max,C}$ = **196.8 MPa (C)**

8.52) ⟨Given⟩ Steel Pipe



$D = 142$ mm

$t = 6.5$ mm

$d = D - 2t = 129$ mm

⟨Goal⟩

Normal stresses at point H and K

⟨Solution⟩

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}\left[(142\text{ mm})^2 - (129\text{ mm})^2\right] = 2{,}766{,}958\text{ mm}^2$$

$$I_z = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}\left[(142\text{ mm})^4 - (129\text{ mm})^4\right] = 6{,}364{,}867\text{ mm}^4$$

$$M_z = (22{,}000\text{ N})(370\text{ mm}) = 8{,}140{,}000\text{ N-mm}$$

$$\sigma_{axial} = \frac{F}{A} = \frac{22{,}000\text{ N}}{2{,}766{,}008\text{ mm}^2} = 7.95\text{ MPa (C)}$$

$$\sigma_{bending} = \frac{M_z c}{I_z} = \frac{(8{,}140{,}000\text{ N-mm})(142\text{ mm}/2)}{6{,}364{,}000\text{ mm}^4} = \pm 90.8\text{ MPa}$$

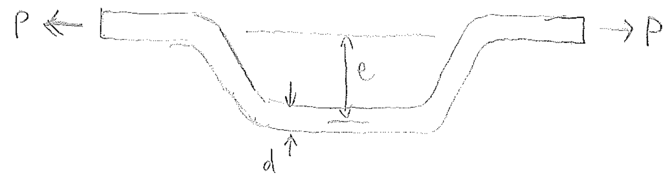
Normal stress at H:

$$\sigma_H = -7.95\text{ MPa} - 90.8\text{ MPa} = -98.8\text{ MPa} = \boxed{98.8\text{ MPa (C)}}$$

Normal stress at K:

$$\sigma_K = -7.95\text{ MPa} + 90.8\text{ MPa} = \boxed{82.9\text{ MPa (T)}}$$

8.54) <Given>



$P = 2500\ N$

$\sigma_{max} = 40\ MPa$

$d = 30\ mm$

<Goal>

Maximum eccentricity for this part

<Solution>

$$A = \frac{\pi}{4}(30\ mm)^2 = 706.9\ mm^2$$

$$I = \frac{\pi}{64}(30\ mm)^4 = 39{,}760.8\ mm^4$$

$$M = (2500\ N)e$$

$$\sigma_{axial} = \frac{F}{A} = \frac{2500\ N}{706.8\ mm^2} = 3.54\ MPa\ (T)$$

$$\sigma_{bending} = \frac{Mc}{I} = \frac{(2500\ N)(30\ mm/2)\ e}{39{,}760.8\ mm^4}$$

$$\sigma_{axial} + \sigma_{bending} \leq 40\ MPa$$

$$3.54\ MPa + \frac{(2500\ N)(30\ mm/2)\ e}{39{,}760.8\ mm^4} \leq 40\ MPa$$

$$e \leq \boxed{38.7\ mm}$$