ENGR 350 - Mechanics of Materials, Fall 2013

Homework #11

Due: Wednesday, Nov. 19

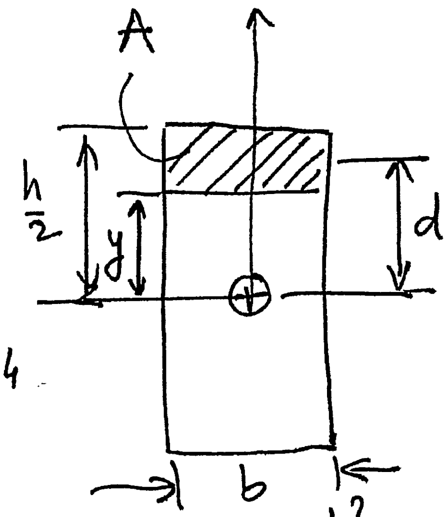
1. Problem P9.9, p. 355
2. Problem P9.12, p. 357
3. Problem P9.22, p. 365
4. Problem P12.2, p. 515
5. Problem P12.14, p. 517

(all problems are from the textbook).

(9.9) Shear force $V = 7.2\ KN$

Shear stress

$$\tau = \frac{VQ}{Ib}$$



$$I = \frac{120 \cdot 280^3}{12} = 219.52 \cdot 10^6\ mm^4$$

$b = 120\ mm$

$$Q = A \cdot d = \frac{1}{2}\left(\frac{h^2}{4} - y^2\right) b$$

Distance from the top

35 mm

$$y = \frac{280}{2} - 35 = 105\ mm \qquad h = 280\ mm$$

$$Q = 514,500\ mm^3$$

$$\tau = \frac{7200 \cdot 514,500}{219.52 \cdot 10^6 \cdot 120} = 140.6 \cdot 10^3\ \frac{N}{m^2}$$

70 mm

$$y = \frac{280}{2} - 70 = 70\ mm \qquad h = 280\ mm$$

$$Q = 882,000\ mm^3$$

$$\tau = \frac{7200 \cdot 882,000}{219.52 \cdot 10^6 \cdot 120} = 241 \cdot 10^3\ \frac{N}{m^2}$$

105 mm $\quad y = \frac{280}{2} - 105 = 35\,mm \quad h = 280\ mm$

$Q = 1{,}102{,}500.0\ mm$

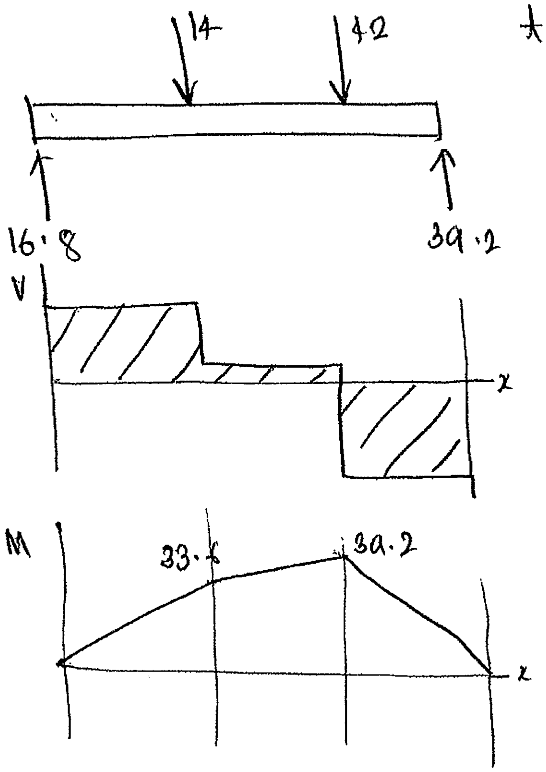
$$\tau = \frac{7200 \cdot 1102500}{219.52 \cdot 10^6 \cdot 120} = 301 \cdot 10^3\ \frac{N}{m^2}$$

140 mm $\quad \tau = \tau_{max} = \frac{3V}{2A} = \frac{3V}{2hb} = \frac{3 \cdot 7200}{2 \cdot 280 \cdot 120} = 321 \cdot 10^3 \frac{N}{m^2}$

Prob 9.12

$$I = \frac{(150\ mm)(450\ mm)^3}{12} = 1139.1\times10^6\ mm^4$$

$t = 150\ mm$



(a) $Q = (150)(150)(150) = 3375000\ mm^3$

$$\tau = \frac{VQ}{It} = \frac{(39200\ N)(3375000\ mm^3)}{(1139.1\times10^6\ mm^4)(150\ mm)} = \boxed{774\ kPa}$$

(b) $Q = (150)(100)(175)$

$= 2625000\ mm^3$

$$\tau = \frac{VQ}{It} = \frac{(39200)(265000)}{(1139.1\times10^6)(150)} = \boxed{602\ kPa}$$

(c) $Q_{max} = (150)(225)(112.5) = 3796875\ mm^3$

$$\tau = \frac{VQ}{It} = \frac{(39200)(3796875)}{(1139.1\times10^6)(150)} = \boxed{871\ kPa}$$

(d) $M_{max} = 39.2\ kN\text{-}m$

$$\sigma_x = -\frac{My}{I} = \frac{(39.2\ kN\text{-}m)(225\ mm)(1000\ N/kN)(1000\ mm/m)}{1139.1\times10^6\ mm^4}$$

$= -7740\ kPa = \boxed{7740\ kPa\ (C)}$

(ANS)

9.22) $I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}((3.5\text{ in})^4 - (3.068\text{ in})^4) = 3.017\text{ in}^4$

$Q = \frac{1}{12}(D^3 - d^3) = \frac{1}{12}((3.5\text{ in})^4 - (3.068\text{ in})^4) = 1.166\text{ in}^3$

$V_{max} = P = 900\text{ lb}$

$M_{max} = 900\text{ lb}(3\text{ft})(12\text{ in/ft}) = 32{,}400\text{ lb-in}$

(a) $\tau = \frac{VQ}{It} = \frac{900\text{ lb}(1.166\text{ in}^3)}{3.017\text{ in}^4(3.5\text{ in} - 3.068\text{ in})} = \boxed{805\text{ psi}}$

(b) $\sigma_x = -\frac{My}{I} = \frac{-(-32{,}400\text{ lb-in})(3.5\text{ in}/2)}{3.017\text{ in}^4} = \boxed{18{,}790\text{ psi (T)}}$

12.2) $A = \frac{\pi}{4}\left[(142\text{ mm})^2 - (128\text{ mm})^2\right] = 2{,}969\text{ mm}^4$

$J = \frac{\pi}{32}\left[(142\text{ mm})^4 - (128\text{ mm})^4\right] = 13{,}563{,}000\text{ mm}^4$

$\sigma = \frac{P}{A} = \frac{(90\text{ KN})(1000\text{ N/KN})}{2{,}969\text{ mm}^2} = 30.3\text{ MPa (T)}$

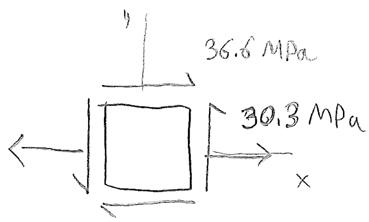
$\tau = \frac{Tc}{J} = \frac{(7\text{ KN-m})(142\text{ mm}/2)(1000\text{ N/KN})(1000\text{ mm/m})}{13{,}563{,}000\text{ mm}^4} = 36.6\text{ MPa}$

Stresses at H:

$\sigma_x = $ 30.3 MPa

$\sigma_y = $ 0 MPa

$\tau_{xy} = $ 36.6 MPa



12.14) $A = \frac{\pi}{4}(1.25\text{ in})^2 = 1.227\text{ in}^2$

$J = \frac{\pi}{32}(1.25\text{ in})^4 = 0.240\text{ in}^4$

$Q = \frac{(1.25\text{ in})^3}{12} = 0.163\text{ in}^3$

$I_y = I_z = \frac{\pi}{64}(1.25\text{ in})^4 = 0.120\text{ in}^4$

$F_x = -520\text{ lb}$ $\quad M_x = 880\text{ lb-in}$

$F_y = 0$ $\quad M_y = -(275\text{ lb})(7\text{ in}) = -1{,}925\text{ lb-in}$

$F_z = 275\text{ lb}$ $\quad M_z = 0$

(a) Point H

$\sigma_x = \frac{520\text{ lb}}{1.227\text{ in}^2} = 424\text{ psi}$

$\tau_{xz} = \frac{275\text{ lb}(0.163\text{ in}^3)}{(0.120\text{ in}^4)(1.25\text{ in})} = 299\text{ psi}$

} Caused by forces

$\tau_{xz} = \frac{M_x c}{J} = \frac{(880\text{ lb-in})(1.25\text{ in}/2)}{0.240\text{ in}^4} = 2{,}295\text{ psi}$

Stresses at H:

$\sigma_x = \boxed{-424\text{ psi}}$

$\sigma_z = 0$

$\tau_{xz} = 299\text{ psi} + 2{,}295\text{ psi} = \boxed{2{,}590\text{ psi}}$

(b) Point K

$\sigma_x = \frac{520\text{ lb}}{1.227\text{ in}^2} = 424\text{ psi}$

$$\tau_{xy} = \frac{M_x c}{J} = \frac{(880 \text{ lb-in})(1.25 \text{ in}/2)}{0.240 \text{ in}^4} = 2{,}295 \text{ psi}$$

$$\sigma_x = \frac{M_y z}{I_y} = \frac{(1{,}925 \text{ lb-in})(1.25 \text{ in}/2)}{0.120 \text{ in}^4} = 10{,}040 \text{ psi}$$

} Caused by moment

Stresses at K:

$\sigma_x = -424 \text{ psi} - 10{,}040 \text{ psi} = \boxed{-10{,}460 \text{ psi}}$

$\sigma_y = 0$

$\tau_{xy} = \boxed{-2{,}290 \text{ psi}}$