ENGR 350 - Mechanics of Materials, Fall 2013

Homework #12

Due: Wednesday, Dec. 10

1. Problem P15.68, p. 677
2. Problem P15.69, p. 677
3. Problem P16.1, p. 691
4. Problem P16.5, p. 691

(all problems are from the textbook).

15.68) $\sigma_{p1,p2} = \frac{30\text{ ksi} + 0\text{ ksi}}{2} \pm \sqrt{\left(\frac{30\text{ ksi} - 0\text{ ksi}}{2}\right)^2 + (21\text{ ksi})^2} = 15\text{ ksi} \pm 25.8\text{ ksi}$

$\sigma_{p1} = 40.8\text{ ksi}$

$\sigma_{p2} = -10.8\text{ ksi}$

(a) Maximum shear stress theory:

$|\sigma_{p1} - \sigma_{p2}| \geq \sigma_y \Rightarrow$ Failure condition

$|40.8\text{ ksi} + 10.8\text{ ksi}| = 51.6\text{ ksi} > 50\text{ ksi} \Rightarrow$ Failure will occur

$FS = \frac{50\text{ ksi}}{51.6\text{ ksi}} = \boxed{0.969}$

(b) Von Mises Shear-Stress Theory

$\sigma_M = (\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2)$

$= ((40.8\text{ ksi})^2 - (40.8\text{ ksi})(-10.8\text{ ksi}) + (-10.8\text{ ksi})^2)^{1/2}$

$= \boxed{47.1\text{ ksi}}$

(c) Maximum-distortion-energy theory factor of safety:

$FS = \frac{50\text{ ksi}}{47.1\text{ ksi}} = \boxed{1.060}$

15.69) $\sigma_{p1,p2} = \frac{15\ ksi - 25\ ksi}{2} \pm \sqrt{\left(\frac{15\ ksi + 25\ ksi}{2}\right)^2 + (-20\ ksi)^2} = -5\ ksi \pm 28.3\ ksi$

$\sigma_{p1} = 23.3\ ksi$

$\sigma_{p2} = -33.3\ ksi$

(a) Maximum-Shear-stress Theory:

$|\sigma_{p1} - \sigma_{p2}| \geq \sigma_y \Rightarrow$ failure condition

$|\sigma_{p1} - \sigma_{p2}| = |23.3\ ksi - (-33.3\ ksi)| = 56.6\ ksi > 50\ ksi \Rightarrow$ component fails

$FS = \frac{50\ ksi}{56.6\ ksi} = \boxed{0.884}$

(b) Mises equivalent stress:

$\sigma_M = (\sigma_{p1}^2 - \sigma_{p1}\sigma_{p2} + \sigma_{p2}^2)^{1/2}$

$= [(23.3\ ksi)^2 - (23.3\ ksi)(-33.3\ ksi) + (-33.3\ ksi)^2]^{1/2}$

$= \boxed{49.2\ ksi}$

(c) Maximum distortion energy theory factor of safety:

$FS = \frac{50\ ksi}{49.3\ ksi} = \boxed{1.015\ ksi} \Rightarrow$ no failure

16.1) (a) $I = \frac{\pi}{64}(16\,mm)^4 = 3,217\ mm^4$

$A = \frac{\pi}{4}(16\,mm)^2 = 201.1\ mm^2$

$r = \sqrt{\frac{3,217\ mm^4}{201.1\ mm^2}} = 4.0\ mm$

$\frac{L}{r} = \frac{1,000\ mm}{4.0\ mm} = \boxed{250}$ ⇒ Slenderness ratio

$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (10,000\ N/mm^2)(3,217\ mm^4)}{(1,000\ mm)^2} = \boxed{318\ N}$

(b) $I = \frac{\pi}{64}(25\,mm)^4 = 19,175\ mm^4$

$A = \frac{\pi}{4}(25\,mm)^2 = 490.9\ mm^2$

$r = \sqrt{\frac{19,175\ mm^4}{490.9\ mm^2}} = 6.25\ mm$

$\frac{L}{r} = \frac{1,000\ mm}{6.25\ mm} = \boxed{160}$

$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (10,000\ N/mm^2)(19,175\ mm^2)}{(1,000\ mm)^2} = \boxed{1,892\ N}$

16.5) Section Properties:

$A = 4.30 \text{ in}^2$

$I_x = 20.9 \text{ in}^4$

$r_x = 2.20 \text{ in}$

$I_y = 11.1 \text{ in}^4$

$r_y = 1.61 \text{ in}$

Euler Buckling Load:

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29{,}000 \text{ ksi})(11.1 \text{ in}^4)}{((24 \text{ ft})(12 \text{ in/ft}))^2} = \boxed{38.3 \text{ Kips}}$$

Allowable Column Load:

$$P_{allow} = \frac{P_{cr}}{FS} = \frac{38.3 \text{ kips}}{1.92} = \boxed{19.95 \text{ Kips}}$$