ENGR 350 - Mechanics of Materials, Fall 2013

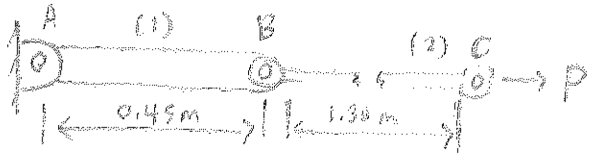
Homework #5

Due: Wednesday, Oct. 8

√ → 1. Problem P5.3, p. 98
2. Problem P5.7, p. 99
3. Problem P5.9, p. 100
4. Problem P5.15, p. 107
√ → 5. Problem P5.25, p. 122
6. Problem P5.29, p. 123
√ → 7. Problem P5.32, p. 124

(all problems are from the textbook).

5.3)



$E_1 = 70$ GPa
$E_2 = 200$ GPa

Elongation;

$$u_C = \delta_1 + \delta_2 = \frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{A_2 E_2}$$

$$F_1 = F_2 = P$$

$$u_C = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}\right]$$

$$A_1 = (30\text{ mm})(80\text{ mm}) = 240\text{ mm}^2$$

$$A_2 = \frac{\pi}{4}(12\text{ mm})^2$$

$$P = \frac{u_C}{\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}}$$

$$= \frac{10\text{ mm}}{\left(\frac{450\text{ mm}}{240\text{ mm}^2 \cdot 70000\text{ N/mm}^2}\right) + \left(\frac{1300\text{ mm}}{113.1\text{ mm}^2\,(200000\text{ N/mm}^2)}\right)}$$

$$\boxed{P = 118.7\text{ KN}}$$

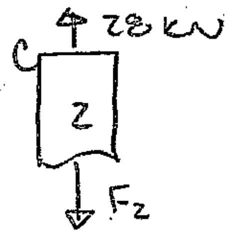
Problem 3.7

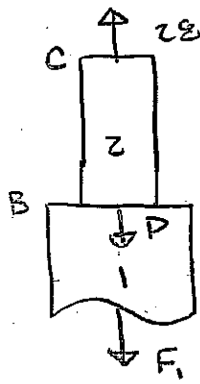
Given: $E = 70\ GPa$ $F = 28KN$

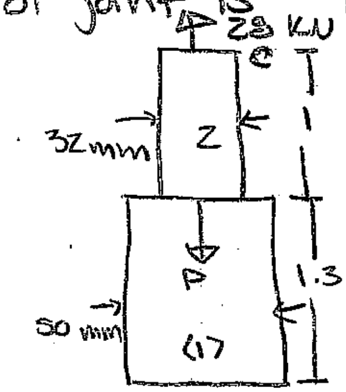
Find: [a] Value of P such that the deflection of joint C is zero.

[b] the corresponding deflection of joint B

Solution:







[a]

$$\Sigma F_y = 0 = 28KN - F_2$$

$$F_2 = 28KN$$

$$\Sigma F_y = 0 = 28\ KN - P - F_1$$

$$F_1 = 28KN - P$$

$$u_c = 0 = \delta_1 + \delta_2$$

$$= \frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{A_2 E_2}$$

$$= \frac{(28000\ N - P)(1000mm)}{\frac{\pi}{4}(50mm)^2(70000\ N/mm^2)} + \frac{28000\ N(1300mm)}{\frac{\pi}{4}(32mm)^2(70000\ N/mm^2)}$$

$$\boxed{P = 80.5841\ KN}$$

[b]

$$u_B = \delta_1 = \frac{F_1 L_1}{A_1 E_1}$$

$$= \frac{(28000\ N - 80584\ N)(1300mm)}{\frac{\pi}{4}(50mm)^2(70000\ N/mm^2)}$$

$$\boxed{u_B = -.497mm}$$

5.9

Given: Tube (1) hollow steel $E = 30{,}000$ KSI

w/ $OD_1 = 2.75$ in $\quad t = 0.25$ in

rod (2)(3) aluminum $E = 10{,}000$ KSI

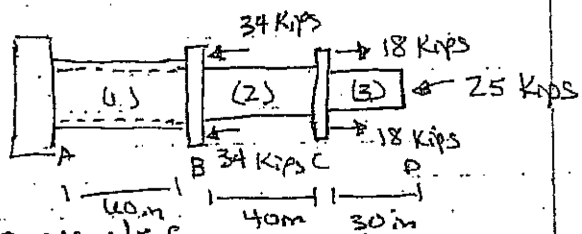
$D_3 = 1.375$ in $\quad D_2 = 2.0$ in

Find:

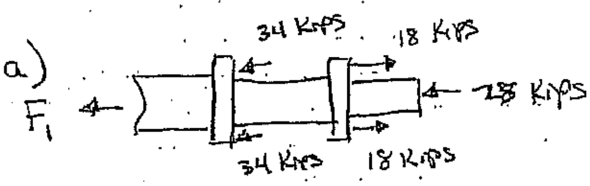
a) ΔL of steel tube

b) δ_D w/ respect to A

c) maximum normal stress in member



a)

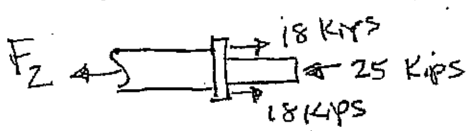


$F_1 = -34 \text{ Kips} - 34 \text{ Kips} + 36 \text{ Kips} - 25 \text{ Kips}$

$F_1 = -57 \text{ Kips}$

$$\delta_1 = \frac{F_1 L_1}{A_1 E_1} = \frac{(-57 \text{ Kip})(60 \text{ in})}{\frac{\pi}{4}(2.75^2 - 2.25^2)\text{ in}^2 \, (30{,}000 \frac{\text{Kip}}{\text{in}^2})} = \boxed{-0.0581 \text{ in}}$$

B) $\delta_D = \delta_1 + \delta_2 + \delta_3$



$F_2 = +18 \text{ Kips} + 18 \text{ Kips} - 25 \text{ Kips}$

$F_2 = 11 \text{ Kips}$

Problem 5.15

Given: Bars 1 and 2 unstressed before P

$E_1 = 100\ GPa$ $A_1 = 520\ mm^2$ $E_2 = 70\ GPa$ $A_2 = 960\ mm^2$

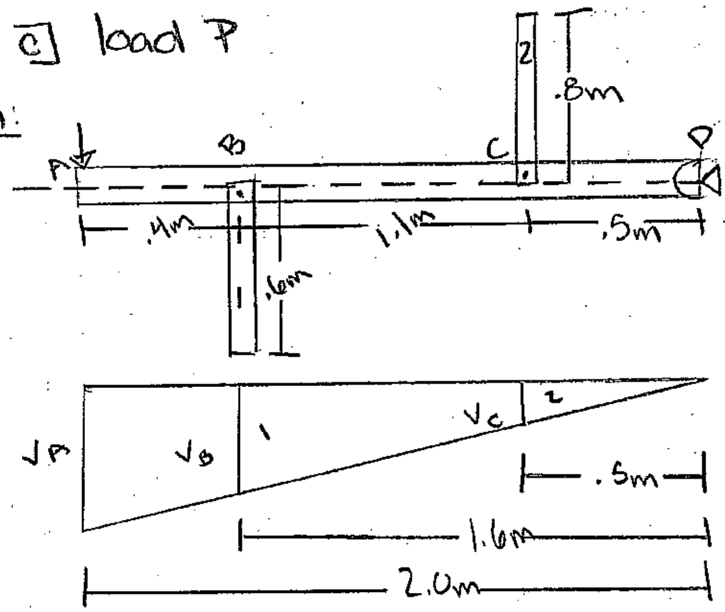
$P_2 = 25\ kN$

Find: [a] stress in 1 and 2

[b] vertical deflection of point A

[c] load P

Solution:



[a]

$$\delta_2 = \frac{F_2 L_2}{A_2 E_2}$$

$$= \frac{(25000\ N)(800\ mm)}{(960\ mm^2)(70{,}000\ N/mm^2)}$$

$$= .297619\ mm$$

$$V_C = \delta_2 = -.297619$$

$$\sigma_2 = \frac{F_2}{A_2}$$

$$= \frac{25000\ N}{960\ mm^2}$$

$$\boxed{\sigma_2 = 26.0416\ MPa}$$

$$\frac{V_B}{1.6m} = \frac{V_C}{.5m}$$

$$V_B = V_C\left(\frac{1.6m}{.5m}\right)$$

$$= 3.2 V_C$$

$$V_B = 3.2(-.297619\ mm)$$

$$V_B = -.952381\ mm$$

$$\delta_1 = V_B = -.952381\ mm$$

$$\delta_1 = \frac{F_1 L_1}{A_1 E_1}$$

$$= \frac{\sigma_1 L_1}{E_1}$$

$$-.952381\,mm = \frac{\sigma_1 (600\,mm)}{100000\,N/mm^2}$$

$$\boxed{\sigma_1 = -158.73016\,MPa}$$

b

$$\frac{V_A}{2m} = \frac{V_c}{.5m}$$

$$V_A = V_c\left(\frac{2m}{.5m}\right)$$

$$= (-.297619)(4)$$

$$\boxed{V_A = -1.190476\,mm}$$

c

$$F_1 = \sigma_1 A_1$$

$$= (-158.730159\,N/mm^2)(520\,mm^2)$$

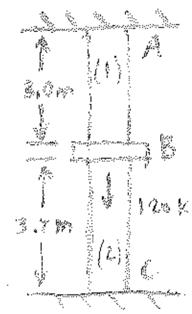
$$= -82539.683\,N$$

$$\Sigma M_D = 0 = 1.6m(F_1) - .5m(F_2) + (2m)P$$

$$P = \frac{.5m(25\,kN) - 1.6m(-82.539\,kN)}{2m}$$

$$\boxed{P = 72.2817\,kN}$$

5.25) <Given>



$A_1 = A_2 = 1475\ mm^2$

$E_1 = E_2 = 200\ GPa$

$u_2 = -1.0\ mm$

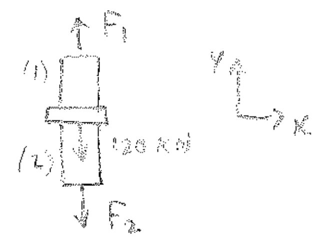
<Goal> At B, find:

(a) normal stresses in upper and lower pipes

(b) Deflection of flange B

<Solution>

(a) $+\uparrow \Sigma F_y: 0 = F_1 - F_2 - 120\ kN$



$\delta_1 + \delta_2 = 1.0\ m$

$$1.0\ mm = \frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{A_2 E_2}$$

$$F_2 = \left(1\ mm - \frac{F_1 L_1}{A_1 E_1}\right)\frac{A_2 E_2}{L_2}$$

$$= \frac{(1\ mm) A_1 E_1}{L_2} - F_1 \frac{L_1 A_2 E_2}{L_2 A_1 E_1}$$

$+\uparrow \Sigma F_y = F_1 - F_2$

$$120\ kN = F_1 - \left(\frac{(1\ mm) A_2 E_2}{L_2} - F_1 \frac{L_1 A_2 E_2}{L_2 A_1 E_1}\right)$$

$F_1 = 110.3\ kN$

$F_2 = F_1 - 120\ kN$

$= 110.3\ kN - 120\ kN$

$= -9.70\ kN$

5.25 cont.)

$$\sigma_1 = \frac{F_1}{A_1}$$

$$= \frac{110300\ N}{1475\ mm^2}$$

$$\boxed{\sigma_1 = 74.8\ MPa\ (T)}$$

$$\sigma_2 = \frac{F_2}{A_2} = \frac{-9700\ N}{1475\ mm^2}$$

$$\boxed{\sigma_2 = -6.58\ MPa\ (C)}$$

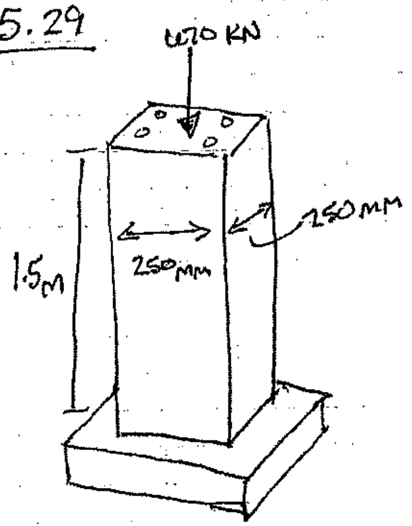
(b) $$u_B = \delta_1$$

$$= \frac{F_1 L_1}{A_1 E_1}$$

$$= \frac{110300\ N\ (3000\ mm)}{1475\ mm^2\ (200000\ N/mm^2)}$$

$$\boxed{u_B = 1.122\ mm\ \downarrow}$$

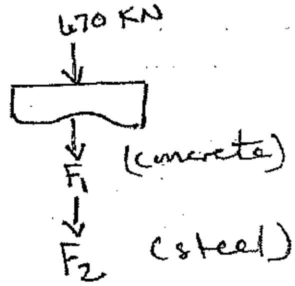
5.29



concrete pier $E = 29$ GPa

4 steel rods $E = 200$ GPa

Determine D of each rod so 20% of load is carried by steel rods



a) $\Sigma F_y = -F_1 - F_2 - 670\ \text{KN}$

Deformation geometry: $\delta_1 = \delta_2$

$$\frac{F_1 L_1}{A_1 E_1} = \frac{F_2 L_2}{A_2 E_2}$$ solve for F_1

$$F_1 = F_2\left(\frac{L_2}{L_1}\right)\left(\frac{A_1}{A_2}\right)\left(\frac{E_1}{E_2}\right)$$

$$\Sigma F_y = -F_1 - F_2 = -\left[F_2\left(\frac{L_2}{L_1}\right)\left(\frac{A_1}{A_2}\right)\left(\frac{E_1}{E_2}\right)\right] - F_2 = 670\ \text{KN}$$

$L_2 = L_1$

$$F_2\left[\left(\frac{A_1}{A_2} \cdot \frac{E_1}{E_2}\right) + 1\right] = -670\ \text{KN}$$

$$F_2 = 0.20\,(-670\ \text{KN}) = \underline{-134\ \text{KN}}$$

$$A_{gross} = 250\ mm^2 = 62{,}500\ mm^2$$

$$A_{gross} = A_1 + A_2 \qquad A_1 = A_{gross} - A_2$$

(concrete) (steel)

$$-134\ KN \left[\frac{62{,}500 - A_2}{A_2} \cdot \frac{29\ GPa}{200\ GPa} + 1 \right] = -670\ KN$$

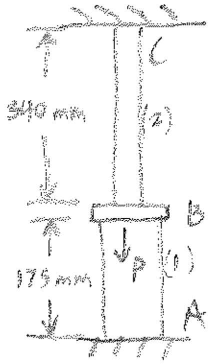
$$\frac{62{,}500\ mm^2 - A_2}{A_2} = 27.586$$

$$A_2 = 2{,}186.37\ mm^2$$

w/ 4 Bars

$$(4)\frac{\pi}{4} d^2_{Bar} \geq 2{,}186.369\ mm^2 \ ; \ \boxed{d_{Bar} \geq 26.4\ mm}$$

5.32) <Given>



$E_1 = 70 \text{ MPa}$

$E_2 = 100 \text{ MPa}$

$d_1 = 35 \text{ mm}$

$d_2 = 20 \text{ mm}$

$\sigma_{allow,1} = 160 \text{ MPa}$

$\sigma_{allow,2} = 110 \text{ MPa}$

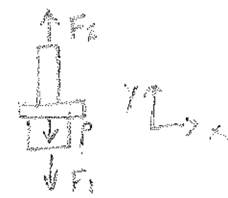
<Goal>

Find:

(a) Max P

(b) u_B due to loading

<Solution>



$+\uparrow \Sigma F_y: 0 = F_2 - F_1 - P$

$\delta_1 + \delta_2 = 0$

$$0 = \frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{A_2 E_2}$$

$$= \frac{\sigma_1 L_1}{E_1} + \frac{\sigma_2 L_2}{E_2}$$

$$\sigma_1 = -\sigma_2 \frac{L_2 E_1}{L_1 E_2}$$

$$= -110 \text{ MPa} \left(\frac{340 \text{ mm} \cdot 70 \text{ GPa}}{175 \text{ mm} \cdot 100 \text{ GPa}} \right)$$

$$= -149.6 \text{ MPa}$$

$$\sigma_2 = -\sigma_1 \frac{L_1 E_2}{L_2 E_1}$$

$$= -160 \text{ MPa} \left(\frac{175 \text{ mm} \cdot 100 \text{ GPa}}{340 \text{ mm} \cdot 70 \text{ GPa}} \right)$$

$$= -117.6 \text{ MPa}$$

$$A_1 = \frac{\pi}{4}(35 \text{ mm})^2 = 962.1 \text{ mm}^2$$

$$A_2 = \frac{\pi}{4}(20 \text{ mm})^2 = 314.2 \text{ mm}^2$$

5.32 cont.)

$F_1 = \sigma_1 A_1$

$= (-149.6\ MPa)(962.1\ mm^2)$

$= -143.9\ KN$

$F_2 = \sigma_2 A_2$

$= 110\ MPa\,(314.2\ mm^2)$

$= 34.6\ KN$

$P_{max} = F_2 - F_1$

$= 34.6\ KN + 143.9\ KN$

$= \boxed{178.5\ KN}$

$v_B = \delta_2$

$= \dfrac{\sigma_2 L_2}{E_2}$

$= \dfrac{110\ MPa\,(340\ mm)}{100{,}000\ MPa}$

$= \boxed{0.374\ mm \downarrow}$