ENGR 350 - Mechanics of Materials, Fall 2013

Homework #6

Due: Monday, Oct. 13

1. Problem P6.3, p. 164
2. Problem P6.5, p. 164
3. Problem P6.8, p. 165
4. Problem P6.10, p. 165
5. Problem P6.13, p. 165
6. Problem P6.14, p. 166

(all problems are from the textbook).

3-0235 — 50 SHEETS — 5 SQUARES
3-0236 — 100 SHEETS — 5 SQUARES
3-0237 — 200 SHEETS — 5 SQUARES
3-0137 — 200 SHEETS — FILLER
COMET

Problem 6.3

Given: hollow steel shaft $D_o = 100\text{ mm}$ $t = 10\text{mm}$

$T = 5500\text{ Nm}$

Find: [a] τ_{max}

[b] minimum diameter so that τ_{max} is same as a

Solution:

[a] $J = \frac{\pi}{32}(D^4 - d^4)$

$= \frac{\pi}{32}(100^4 - 80^4)$

$= 5796238\text{ mm}^4$

$\tau_{max} = \frac{Tc}{J}$

$= \frac{(5500\text{ Nm})(100\text{mm}/2)(1000\text{ mm/m})}{5796238\text{ mm}^4}$

$= 47.445\text{ MPa}$

[b] $J = \frac{\pi}{32}d^4$

$\tau = \frac{Tc\,32}{\pi d^4}$

$\frac{\pi d^{4\,3}}{16\,32\,d} = \frac{T}{\tau}$

$d^3 = \frac{T16}{\pi\tau}$

$d^3 = \frac{5500(1000)(16)}{\pi(47.445)}$

$d^3 = 590394.56\text{ mm}^3$

$d = 83.891\text{ mm}$

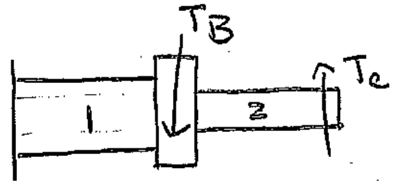
Problem 6.5

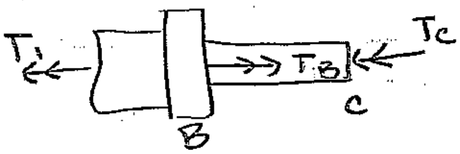
Given: $d_{o1} = 10.75 in$ $t_1 = .365 in$ $d_{o2} = 6.625 in$

$t_2 = .280$ in $T_B = 60$ kipft $T_C = 24$ kipft

Find: τ_{max} in each shaft

Solution:





$\Sigma M_x = 0$

$= -T_1 + 60 - 24$

$T_1 = 36$ kipft

$\Sigma M_x = 0$

$= -T_2 - 24$

$T_2 = -24$ kipft

$$J_1 = \frac{\pi}{32}\left[D^4 - d^4\right]$$

$$= \frac{\pi}{32}\left(10.75^4 - 10.02^4\right)$$

$$= 321.4685 \text{ in}^4$$

$$J_2 = \frac{\pi}{32}\left(6.625^4 - 6.065^4\right)$$

$$J_2 = 56.2844 \text{ in}^4$$

$$\tau_1 = \frac{Tc}{J}$$

$$= \frac{36 \text{ kipft}(10.75/2 \text{ in})(12 \text{ in/ft})}{321.4685 \text{ in}^4}$$

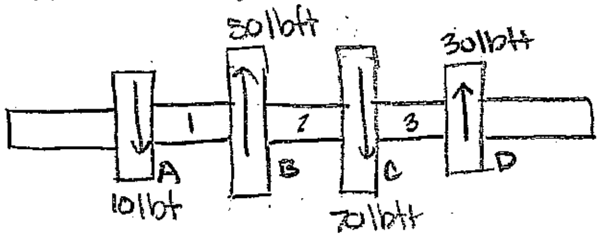
$$\boxed{\tau_1 = 7.22 \text{ ksi}}$$

$$\tau_2 = \frac{(24 \text{ kipft})(12 \text{ in/ft})(6.625/2 \text{ in})}{56.2844 \text{ in}^4}$$

$$\boxed{\tau_2 = 16.95 \text{ ksi}}$$

Problem 6.8

Given: $d = .75$ in

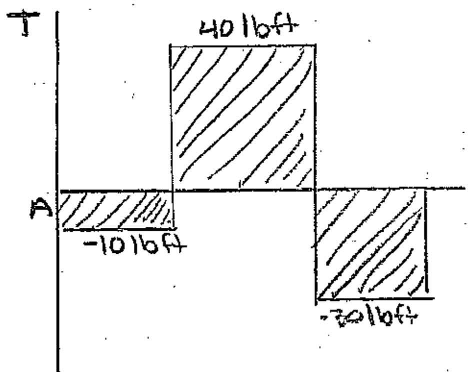


Find: a) torque diagram

b) τ_{max} in shaft

Solution:

a)

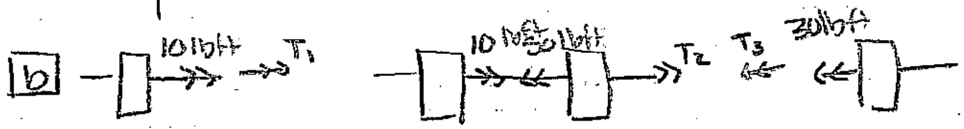


$J = \frac{\pi}{32} d^4$

$= \frac{\pi}{32}(.75)^4$

$= .031063 \text{ in}^4$

b)



$\Sigma M_x = 0 = T_1 + 10$

$T_1 = -10 \text{ lbft}$

$\Sigma M_x = 0 = T_2 + 10 \text{ lbft} - 50 \text{ lbft}$

$T_2 = 40 \text{ lbft}$

$\Sigma M_x = 0 = -T_3 - 30 \text{ lbft}$

$T_3 = -30 \text{ lbft}$

$\tau_1 = \frac{T_1 c_1}{J}$

$= \frac{-10 \text{ lbft}(12 \text{ in/ft})(.75/2 \text{ in})}{.031063 \text{ in}^4}$

$\tau_1 = 1448.7 \text{ psi}$

$\tau_2 = \frac{(40 \text{ lbft})(12 \text{ in/ft})(.75/2 \text{ in})}{.031063 \text{ in}^4}$

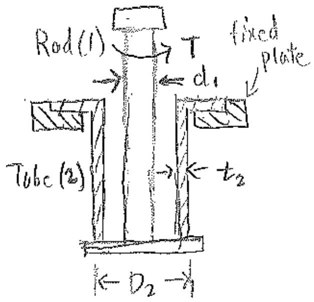
$\tau_2 = 5794.7 \text{ psi}$

$\tau_3 = \frac{(30 \text{ lbft})(12 \text{ in/ft})(.75/2 \text{ in})}{.031063 \text{ in}^4}$

$\tau_3 = 4346.0 \text{ psi}$

$\tau_{max} = 5794.7 \text{ psi}$

6.10) <Given>



$\tau_{allow,1} = 18$ ksi

$\tau_{allow,2} = 6$ ksi

$D_2 = 1.50$ in

$t_2 = 0.125$ in

<Goal>

Calculate:

(a) Largest T for the allowed shear stress on tube (2)

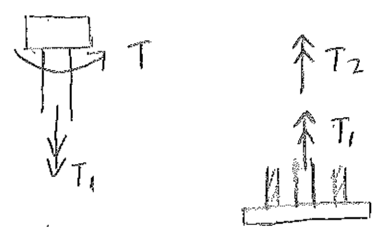
(b) minimum d_1

<Solution>

$T_1 = T$

$\Sigma M: 0 = T_1 + T_2$

$T_2 = -T_1 = -T$



(a) $J_2 = \frac{\pi}{32}\left[(1.50\text{in})^4 - (1.25\text{in})^4\right]$

$= 0.257 \text{ in}^4$

$T_2 = \frac{\tau_{allow,2} J_2}{c_2}$

$= \frac{6000 \text{ psi} (0.257 \text{ in}^4)}{0.75 \text{ in}}$

$= \boxed{2.06 \text{ kip-in}}$

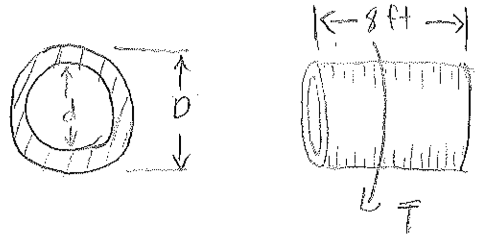
(b) $\frac{\pi}{16} d_1^3 \geq \frac{T}{\tau}$

$d_1^3 = \frac{16T}{\pi\tau}$

$= \frac{16(2060{,}000 \text{ lb-in})}{\pi(18{,}000 \text{ lb/in}^2)}$

$d \doteq \boxed{0.835 \text{ in}}$

6.13) <Given>



$D = 3.5\ in$

$T = 3750\ lb\text{-}ft$

$G = 12{,}000\ ksi$

$\phi \geq 3^\circ$ for the full length

<Goal>

Find d and report τ and ϕ

<Solution> Check shear stress:

$$J = \frac{\pi}{32}(D^4 - d^4)$$

$$\frac{\pi}{32}\frac{(D^4 - d^4)}{D/2} = \frac{T}{\tau}$$

$$D^4 - d^4 \geq \frac{32TD}{2\pi\tau}$$

$$d^4 \leq D^4 - \frac{32TD}{2\pi\tau}$$

$$d \leq \sqrt[4]{(3.5\ in)^4 - \frac{32(3750\ lb\text{-}ft)(3.5\ in)(12\ in/ft)}{2\pi(8000\ psi)}}$$

$$d \leq 2.656\ in$$

Check angle of twist:

$$\phi = \frac{TL}{JG}$$

$$J = \frac{\pi}{32}(D^4 - d^4)$$

$$\frac{\pi}{32}(D^4 - d^4) \geq \frac{TL}{\phi G}$$

$$d^4 \leq D^4 - \frac{32TL}{\pi\phi G}$$

$$d \leq \sqrt[4]{(3.5\ in)^4 - \frac{32(3750\ lb\text{-}ft)(8\ ft)(12\ in/ft)^2}{\pi(3^\circ)(\pi/180^\circ)(12{,}000{,}000\ psi)}}$$

$$d \leq \boxed{2.66\ in}$$

6.13 cont.)

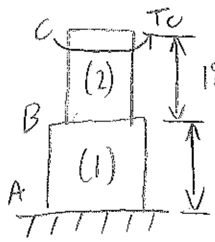
$$\phi = \frac{TL}{JG}$$

$$= \frac{3750 \text{ lb-ft} (8 \text{ ft})(12 \text{ in/ft})^2}{9.817 \text{ in}^4 (12{,}000{,}000 \text{ psi})}$$

$$= 0.0367 \text{ rad} \left(\frac{180}{\pi}\right)\frac{\text{deg}}{\text{rad}}$$

$$= \boxed{2.10^\circ}$$

6.14) ⟨Given⟩



$G_1 = 5600$ ksi
$d_1 = 1.75$ in
$\tau_{allow,1} = 9000$ psi
$G_2 = 4000$ ksi
$d_2 = 1.25$ in
$\tau_{allow,2} = 12000$ psi

⟨Goal⟩

Find max T_C if $\phi_C \leq 4°$

⟨Solution⟩

$$J_1 = \frac{\pi}{32}(1.75 \text{ in})^4 = 0.921 \text{ in}^4$$

$$T_1 \leq \frac{\tau_1 J_1}{c_1} \leq \frac{9000 \text{ psi } (0.921 \text{ in}^4)}{(1.75 \text{ in}/2)} \leq 9471 \text{ lb-in}$$

$$J_2 = \frac{\pi}{32}(1.25 \text{ in})^4 = 0.2397 \text{ in}^4$$

$$T_2 \leq \frac{\tau_2 J_2}{c_2} \leq \frac{12000 \text{ psi } (0.2397 \text{ in}^4)}{(1.25 \text{ in}/2)} \leq 4602 \text{ lb-in}$$

$$\phi_C = \phi_1 + \phi_2 = \frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2} \quad ; \quad T_1 = T_2 = T_C$$

6.14 cont.)

$$\phi_C = T_C\left(\frac{L_1}{J_1G_1} + \frac{L_2}{J_2G_2}\right)$$

$$T_C \le \frac{\phi_C}{\frac{L_1}{J_1G_1} + \frac{L_2}{J_2G_2}}$$

$$\le \frac{4^{\circ}\left(\frac{\pi}{180}\right)\frac{rad}{deg}}{\frac{12\,in}{(0.921\,in^4)(5{,}600{,}000\,psi)} + \frac{18\,in}{(0.2397\,in^4)(4{,}000{,}000\,psi)}}$$

$$\le \boxed{3{,}308.4\ lb\text{-}in}$$