ENGR 350 - Mechanics of Materials, Fall 2013

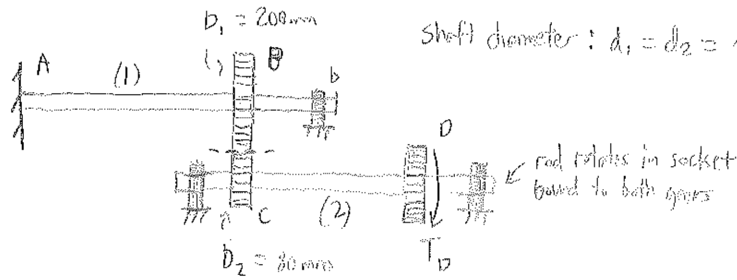
Homework #7

Due: Friday, Oct. 17

1. Problem P6.27, p. 174
2. Problem P6.33, p. 175
3. Problem P6.36, p. 179
4. Problem P6.38, p. 179
5. Problem P6.67, p. 197

(all problems are from the textbook).

6.27) ⟨Given⟩



$\tau_{1,allow} = \tau_{2,allow} = 50$ MPa

Shaft diameter : $d_1 = d_2 = 20$ mm

⟨Goal⟩

Find maximum T_D

⟨Solution⟩

$$J_1 = J_2 = \frac{\pi}{32}(20\text{ mm})^4 = 15.71 \times 10^6 \text{ mm}^4$$

$$T_{max} = \frac{\tau_{allow} J_1}{c_1}$$

$$= \frac{50 \text{ N/mm}^2 (15.71 \times 10^6 \text{ mm}^4)}{10 \text{ mm}}$$

$$= 78,540 \text{ N-mm}$$

$$\frac{T_1}{200\text{mm}} = \frac{T_2}{80\text{ mm}}$$

$T_1 = 2.5\, T_2$; T_1 is the determinant

$T_1 = 78,540$ N-mm

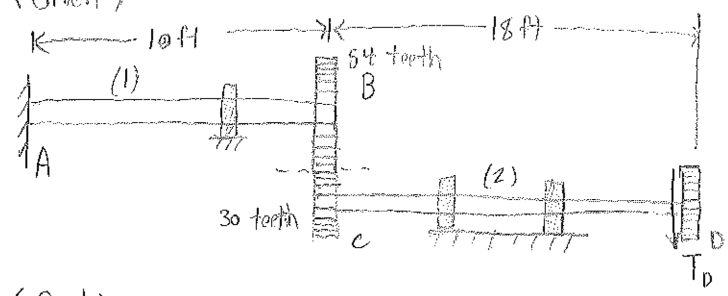
$$T_2 = \frac{T_1}{2.5}$$

$$= 31,416 \text{ N-mm}$$

$T_2 = T_D$

$$T_D = \boxed{31.4 \text{ N-m}}$$

6.33) ⟨Given⟩



shaft diameter: $d_1 = d_2 = 2''$

$G = 12{,}000$ ksi

$\phi_D = 6°$

⟨Goal⟩

Find T_{max} for each shaft

⟨Solution⟩

$$\Sigma M_x : 0 = T_D - T_2 \quad ; \quad T_2 = T_D$$

$$T_C = T_2 = T_D$$

$$T_B = -T_C \frac{54 \text{ teeth}}{30 \text{ teeth}} = -1.8\,T_C$$

$$T_1 = -1.8\,T_2$$

$$\phi_1 = \phi_B - \phi_A$$

$$\phi_2 = \phi_D - \phi_C$$

$$\phi_C = -\phi_B \frac{54 \text{ teeth}}{30 \text{ teeth}}$$

$$\phi_D = \phi_2 + \phi_C$$

$$= \phi_2 - \phi_B \cdot \frac{54 \text{ teeth}}{30 \text{ teeth}}$$

$$= \phi_2 - \phi_1 \frac{54 \text{ teeth}}{30 \text{ teeth}} = 6°$$

$$\phi_D = \frac{T_2 L_2}{J_2 G_2} - 1.8 \frac{T_1 L_1}{J_1 G_1} = 6°\left(\frac{\pi}{180}\right) = 0.1047 \text{ rad}$$

$$0.1047 \text{ rad} = \frac{T_2}{JG}\left(L_2 + 1.8^2 L_1\right) \quad ; \quad J = \frac{\pi}{32}(2\text{in})^4 = 1.571 \text{ in}^4$$

$$T_2 = \frac{0.1047 \text{ rad}(1.571 \text{ in}^4)(12{,}000{,}000 \text{ psi})}{[18 \text{ ft} + 1.8^2(10 \text{ ft})](12 \text{ in/ft})}$$

$$= 3{,}264 \text{ lb-in}$$

6.33 cont.) $T_1 = -1.8\,T_2$

$= -1.8(3{,}264 \text{ lb-in})$

$= -5{,}875 \text{ lb-in}$

$$\tau_1 = \frac{T_1 c_1}{J_1}$$

$$= \frac{5875 \text{ lb-in } (1 \text{ in})}{1.571 \text{ in}^4}$$

$$= \boxed{3740 \text{ psi}}$$

$$\tau_2 = \frac{T_2 c_2}{J_2}$$

$$= \frac{3264 \text{ lb-in } (1 \text{ in})}{1.571 \text{ in}^4}$$

$$= \boxed{2080 \text{ psi}}$$

6.36) <Given>

Driveshaft of an automobile is being designed to transmit 180 hp at 3,500 rpm. τ_{shaft} cannot exceed 6,000 psi.

<Goal>

d_{min} of the shaft

<Solution>

$$T = \frac{P}{\omega}$$

$$= \frac{180 \text{ hp}}{\frac{3500 \text{ rev}}{\text{min}}} \Rightarrow \text{ideal units: lb-ft}$$

$$= \frac{180 \text{ hp}\left(\frac{550 \frac{\text{lb-ft}}{\text{s}}}{1 \text{ hp}}\right)}{\frac{3500 \text{ rev}}{1 \text{ min}}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)}$$

$$= 270.1 \text{ lb-ft}$$

Conversions

$1 \text{ hp} = 550 \frac{\text{lb-ft}}{\text{s}}$

$1 \text{ revolution} = 2\pi \text{ rad}$

$1 \text{ min} = 60 \text{ s}$

$$\frac{\pi}{16} d^3 \geq \frac{T}{\tau_{allow}}$$

$$\geq \frac{270.1 \text{ lb-ft}\,(12 \text{ in/ft})}{6000 \text{ psi}}$$

$$\geq 0.5402 \text{ in}^3$$

$$d \geq \boxed{1.401 \text{ in}}$$

6.38

$\tau_{allow} = 30\ \text{MPa}$ $\quad P = 225\ \text{KW}$ $\quad \omega = 1700\ \text{rpm}$

$OD = 75\ \text{mm}$: Determine tubular wall thickness

Solution:

$$T = \frac{P}{\omega} = \frac{(225\ \text{KW})\left(1000\ \frac{\text{N-M}}{\text{KW}^{s}}\right)}{\left(1700\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\text{s}}\right)} = \underline{\underline{1{,}263.88\ \text{N-M}}}$$

$$\tau = \frac{Tc}{J}\ ;\ J = \frac{\pi}{32}\left[D^4 - d^4\right]$$

$$\frac{\pi}{32}\frac{\left[D^4 - d^4\right]}{D/2} \geq \frac{T}{\tau}$$

$$\frac{\pi}{32}\frac{\left[75^4\ \text{mm}^4 - d^4\right]}{\frac{75}{2}\ \text{mm}} \geq \frac{(1{,}263.877\ \text{N-m})(1{,}000\ \text{mm/m})}{30\ \text{N/mm}^2}$$

solve for d

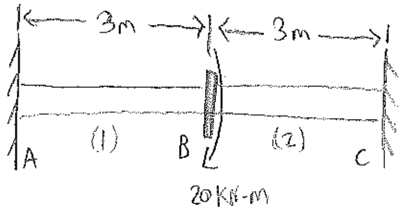
$$d \simeq 62.7945\ \text{mm}$$

minimum thickness

$$D = d + 2t$$

$$t \geq \frac{D - d}{2} = \frac{75\ \text{mm} - 62.7945\ \text{mm}}{2} = \boxed{6.103\ \text{mm}}$$

6.67) <Given>



$D_1 = 168\,mm$

$d_1 = 7\,mm$

$D_2 = 114\,mm$

$d_2 = 6\,mm$

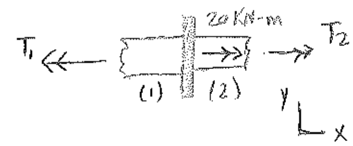
$G_1 = G_2 = 80\,GPa$

<Goal>

(a) τ_{max} for (1) and (2)

(b) ϕ_b relative to A

<Solution>



$\sum M_x : 0 = -T_1 + T_2 + 20\ KN\text{-}m$

$\phi_1 + \phi_2 = 0$

$0 = \frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2}$

$T_1 = -\frac{T_2 L_2 J_1 G_1}{L_1 J_2 G_2}$

$= -T_2 \left[\frac{3\cancel{m}\,(22{,}987{,}000\ mm^4)\,(80\,\cancel{GPa})}{3\cancel{m}\,(5{,}954{,}000\ mm^4)\,(80\,\cancel{GPa})}\right]$

$\doteq -3.8604\ T_2$

$J_1 = \frac{\pi}{32}\left[(168\,mm)^4 - (154\,mm)^4\right]$

$= 22{,}987{,}000\ mm^4$

$J_2 = \frac{\pi}{32}\left[(114\,mm)^4 - (102\,mm)^4\right]$

$= 5{,}955{,}000\ mm^4$

$-T_1 + T_2 = -20\ KN\text{-}m$

$4.8604\ T_2 = -20\,KN\text{-}m$

$T_2 = -4.1149\ KN\text{-}m$

$T_1 = T_2 + 20\ KN\text{-}m$

$= 15.8851\ KN\text{-}m$

(a) $\tau_1 = \frac{T_1 C_1}{J_1}$

$= \frac{15.8851\ KN\text{-}m\ (168\,mm/2)(1000)^2}{22{,}987{,}000\ mm^4}$

$= \boxed{58.0\ MPa}$

6.67 cont) $\tau_2 = \frac{T_2 c_2}{J_2}$

$= \frac{4.1149 \text{ KN-m } (114\text{mm}/2)(1000)^2}{5{,}954{,}000 \text{ mm}^4}$

$= \boxed{39.4 \text{ MPa}}$

(b) $\phi_1 = \phi_B - \phi_A$; $\phi_A = 0$; $\phi_B = \phi_1$

$\phi_1 = \frac{T_1 L_1}{J_1 G_1}$

$= \frac{15.8851 \text{ KN-m } (3{,}000\text{mm})(1000 \text{ mm/m})(1000 \text{ N/KN})}{22{,}987{,}000 \text{ mm}^4 (80{,}000 \text{ N/mm}^2)}$

$= \boxed{0.0259 \text{ rad}}$