ENGR 350 - Mechanics of Materials, Fall 2013

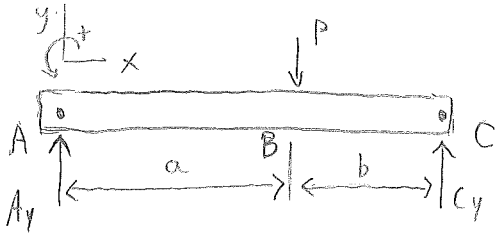
Homework #8

Due: Friday, Oct. 24

1. Problem P7.2, p. 225
2. Problem P7.4, p. 225
3. Problem P7.6, p. 225
4. Problem P7.7, p. 225
5. Problem P7.8, p. 225

(all problems are from the textbook).

7.2) <Given> Simply-supported beam



<Goal>

(a) Derive equations for shear force and bending moment across the beam

(b) Create a shear and moment diagram for the beam

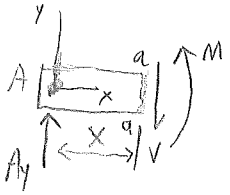
<Solution>

$$+\uparrow \Sigma F_y : 0 = A_y + C_y - P$$

$$\circlearrowleft^{+} \Sigma M_A : 0 = -P \cdot a + C_y (a+b)$$

$$A_y = \frac{P \cdot b}{a+b}$$

$$C_y = \frac{P \cdot a}{a+b}$$



Section a-a: $0 \leq x \leq a$

$$\Sigma F_y : 0 = A_y - V$$

$$= \frac{P \cdot b}{a+b} - V$$

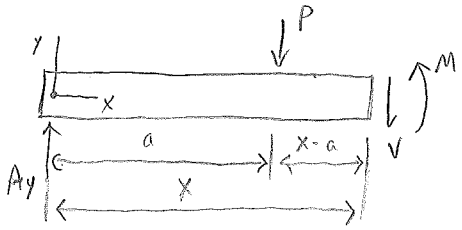
$$\boxed{V = \frac{P \cdot b}{a+b}}$$

$$\Sigma M_{a\text{-}a} : 0 = -A_y \cdot x + M$$

$$= -\left(\frac{P \cdot b}{a+b}\right) \cdot x + M$$

$$\boxed{M = \left(\frac{P \cdot b}{a+b}\right) \cdot x}$$

7.2 cont.) Section b-b: $a \leq x \leq a+b$

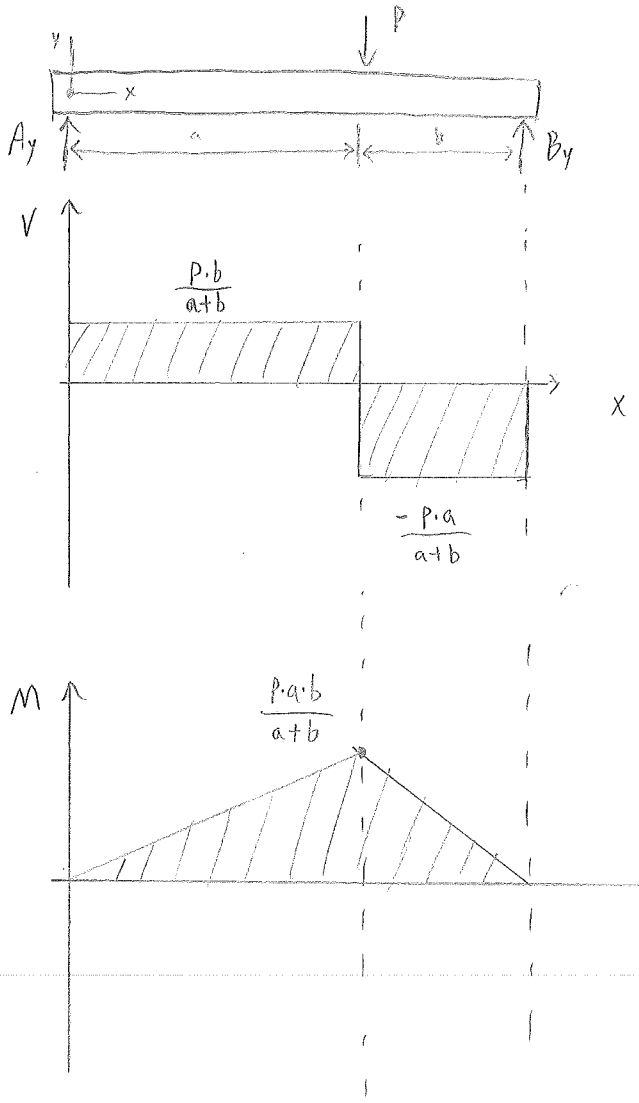


$$+\uparrow \Sigma F_y : 0 = A_y - P - V$$

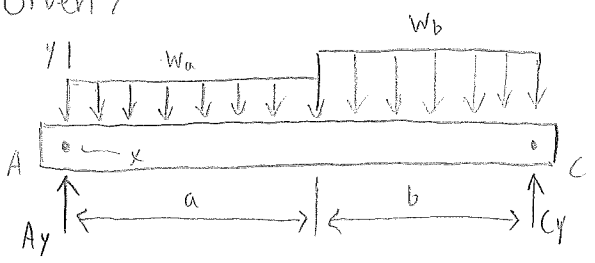
$$\boxed{V = -\left(\frac{P \cdot a}{a+b}\right)}$$

$$\circlearrowleft \Sigma M_{b\text{-}b} : 0 = -A_y \cdot x + P(x-a) + M$$

$$\boxed{M = \left(\frac{P \cdot b}{a+b}\right)x - P(x-a)}$$



7.4) <Given>



<Goal>

(a) Desive V, M equations

(b) Plot shear-force and bending-moment diagrams

<Solution>

Equilibrium:

$$\circlearrowleft \Sigma M_C : 0 = w_a \cdot a\left(b + \frac{a}{2}\right) + w_b \cdot b\left(\frac{b}{2}\right) - A_y(a+b)$$

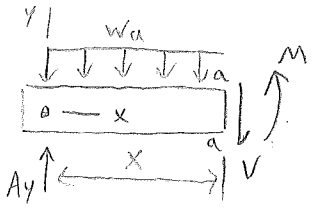
$$A_y = \frac{w_a a(a+2b) + w_b b^2}{2(a+b)}$$

$$\circlearrowleft \Sigma M_A : 0 = -w_a a\left(\frac{a}{2}\right) - w_b b\left(a + \frac{b}{2}\right) + C_y(a+b)$$

$$C_y = \frac{w_a a^2 + w_b b(2a+b)}{2(a+b)}$$

Section a-a: $0 \le x \le a$

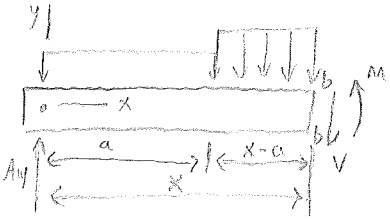
a-a:



$$+\uparrow \Sigma F_y : 0 = A_y - w_a x - V$$

$$\boxed{V = \frac{w_a a(a+2b) + w_b b^2}{2(a+b)} - w_a x}$$

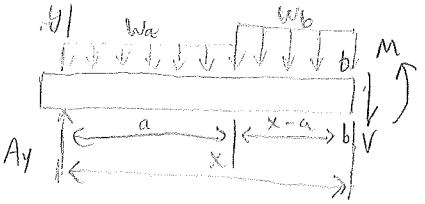
b-b:



$$\circlearrowleft \Sigma M_{a-a} : 0 = -A_y x + w_a x\left(\frac{x}{2}\right) + M$$

$$\boxed{M = \left(\frac{w_a a(a+2b) + w_b b^2}{2(a+b)}\right)x^2 - \frac{w_a x^2}{2}}$$

7.4 cont.) Section b-b':



$$+\uparrow \Sigma F_y : 0 = A_y - w_a \cdot a - w_b(x-a) - V$$

$$V = A_y - w_a a - w_b(x-a)$$

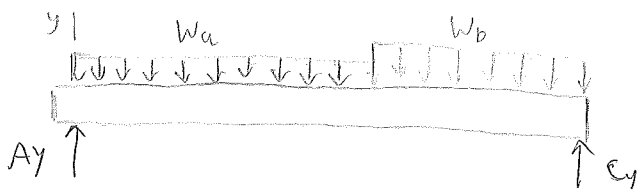
$$= \boxed{\frac{w_a a(a+2b)}{2(a+b)} + \frac{w_b b^2}{2(a+b)} - w_a a - w_b(x-a)}$$

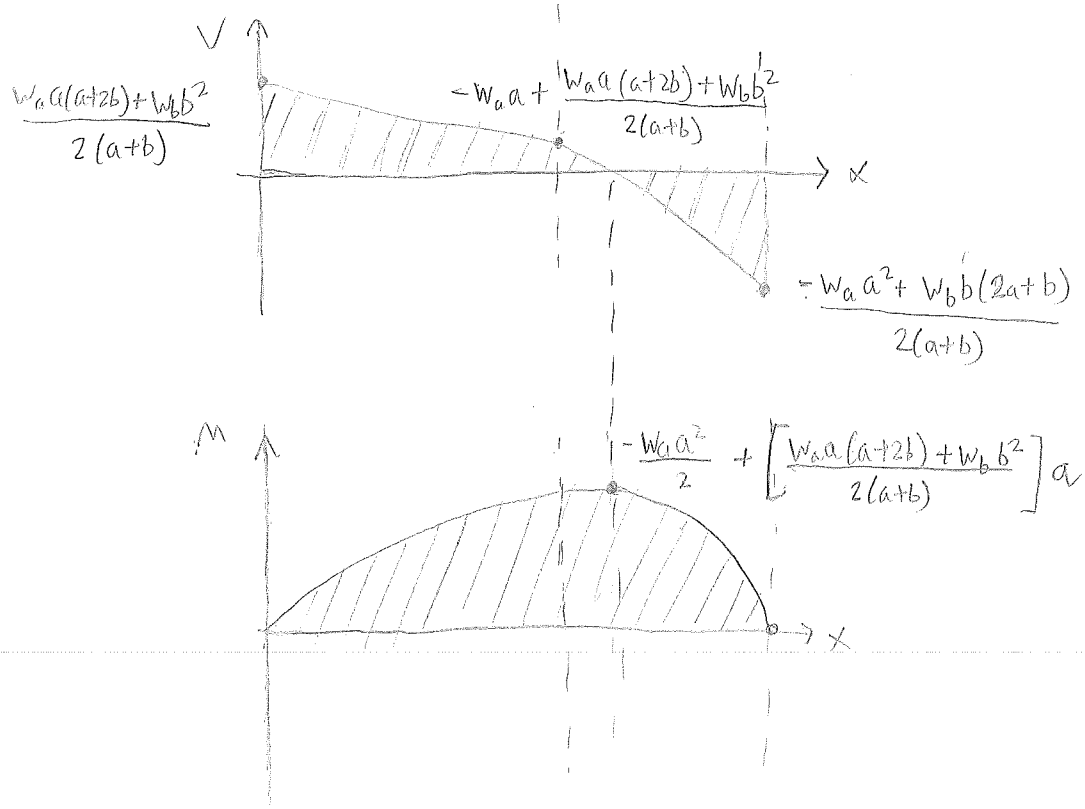
$$\circlearrowleft \Sigma M_{b-b'} : 0 = -A_y x + w_a a\left(x - \frac{a}{2}\right) + w_b(x-a)\left(\frac{x-a}{2}\right) + M$$

$$M = A_y x - w_a a\left(x - \frac{a}{2}\right) - w_b \frac{(x-a)^2}{2}$$

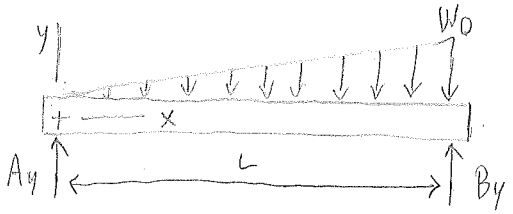
$$= \left[\frac{w_a a(a+2b)}{2(a+b)} + \frac{w_b b^2}{2(a+b)}\right] x - w_a a\left(x - \frac{a}{2}\right) - w_b \frac{(x-a)^2}{2}$$

(b)





7.6) ⟨Given⟩



⟨Goal⟩

(a) Derive equations for V and M

(b) Shear and bending diagrams

(c) Find M_{max} and its location

⟨Solution⟩

$$\circlearrowleft + \Sigma M_B : 0 = -A_y L + \frac{W_0 L}{2}\left(\frac{L}{3}\right)$$

$$A_y = \frac{W_0 L}{6}$$

$$\circlearrowleft + \Sigma M_A : 0 = B_y L - \frac{W_0 L}{2}\left(\frac{2L}{3}\right)$$

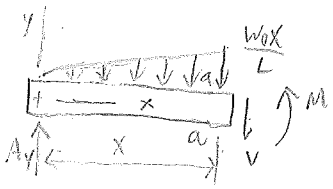
$$B_y = \frac{W_0 L}{3}$$

(a) $$+\uparrow \Sigma F_y : 0 = A_y - \frac{W_0 x}{L}\left(\frac{x}{2}\right) - V$$

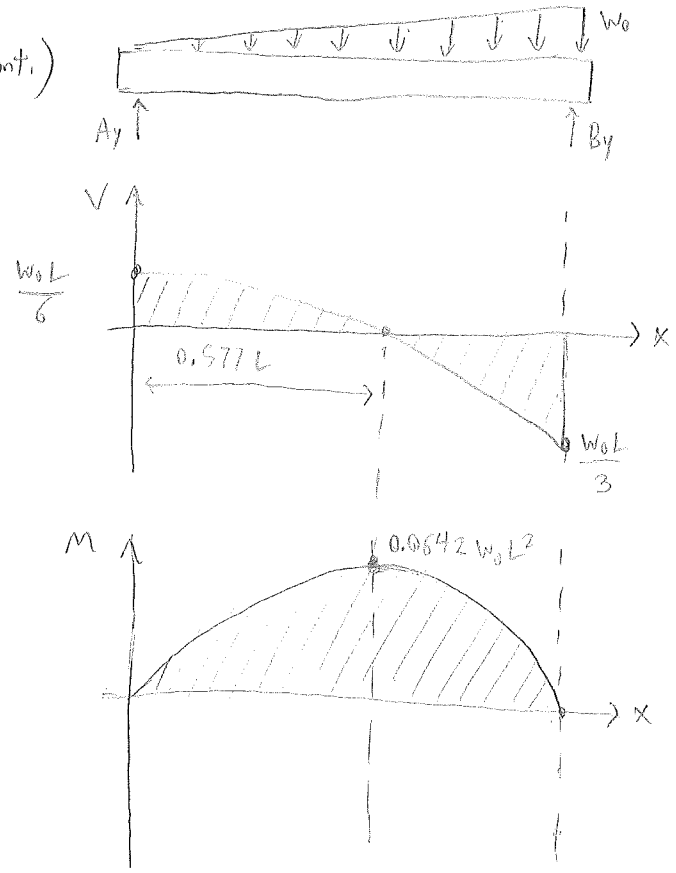
$$\boxed{V = \frac{W_0 L}{6} - \frac{W_0 x^2}{2L}}$$

$$\circlearrowleft + \Sigma M_{a-a} : 0 = -A_y x + \frac{W_0 x}{L}\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) + M$$

$$\boxed{M = -\frac{W_0 x^3}{6L} + \frac{W_0 L x}{6}}$$



7.6 cont.)



(c) $V = \frac{w_0 L}{6} - \frac{w_0 x^2}{2L} = 0$

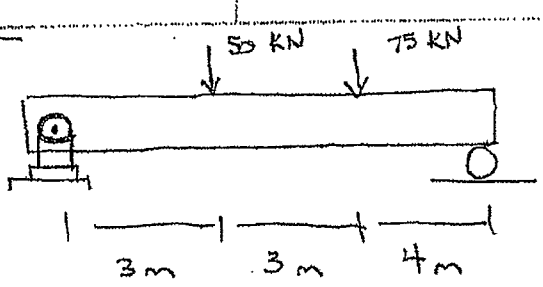
$x = \sqrt{\frac{L^2}{3}} = \frac{L}{\sqrt{3}} = \boxed{0.577L}$

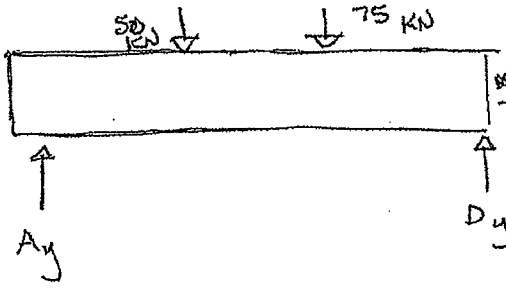
$M = -\frac{w_0 x^3}{6L} + \frac{w_0 L x}{6}$

$M_{max} = -\frac{w_0 (0.577L)^3}{6L} + \frac{w_0 L (0.577L)}{6}$

$= \boxed{0.0642\, w_0 L^2}$

7.7





Equilibrium

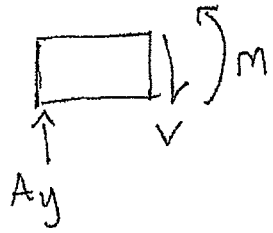
$\overset{+}{\circlearrowleft} \Sigma M_A = -50\,KN(3m) - 75\,KN(6m) + D_y(10m)$

$D_y = 60\ KN$

$\Sigma F_y = A_y + D_y - 50\,KN - 75\,KN$

$A_y = 65\ KN$

a-a



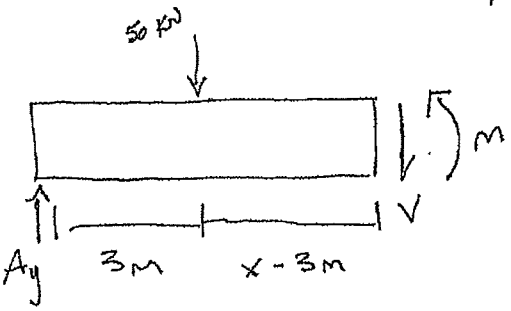
$\Sigma F_y \Rightarrow A_y - V = 0$

$V = 65\ KN$

$\Sigma M_{a-a} = -A_y x + M$

$M = 65(x)\ KN\text{-}m$

B-B



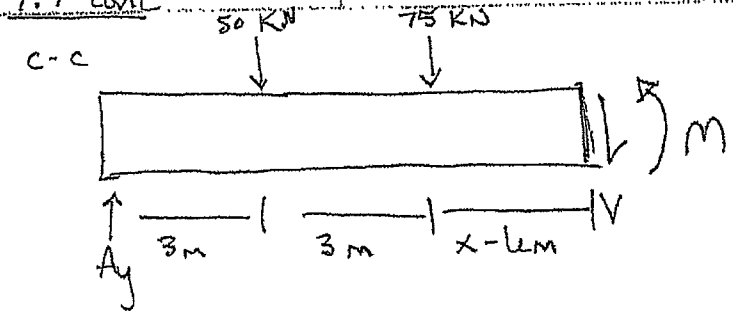
$\Sigma F_y = -V + A_y - 50\,KN = 65\,KN - 50\,KN = 15\,KN = V$

$\overset{+}{\circlearrowleft} \Sigma M_{b-b} = -A_y x + 50\,KN(x-3)m + M$

$= -65x + 50x\,KN\text{-}m - 150\,m + M$

$M = 15x - 150\ KN\text{-}m$

7.7 cont



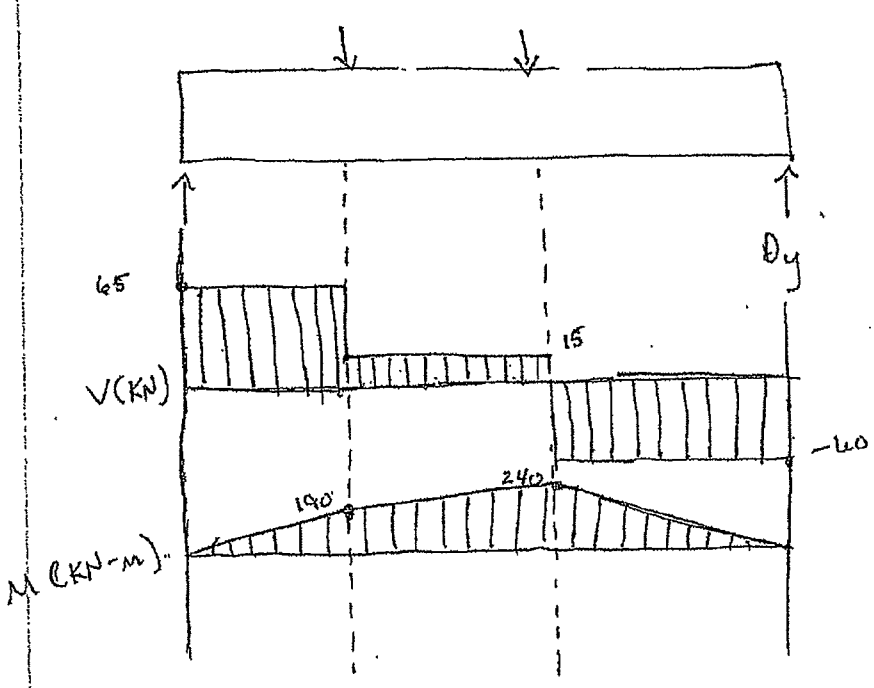
$\Sigma F_y = A_y - 50\,KN - 75\,KN - V$

$V = -60\,KN$

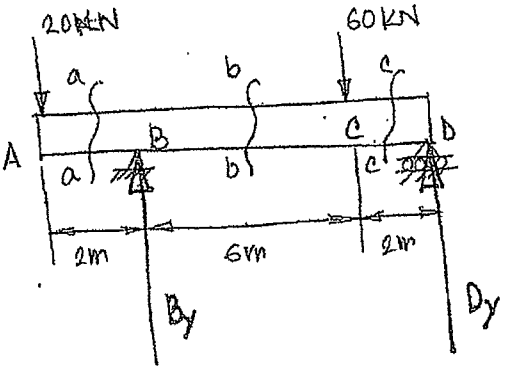
$(+\circlearrowleft) \Sigma M_{cc} = (50\,KN)(x-3m) + 75\,KN \cdot (x-6)m + M + A_y x$

$0 = +150\,KN\cdot m + 50\,KN\cdot x\,m + 75\,KN(x\,m) - 450\,KN\cdot m + M$

$M = -60\,KN\cdot x\,m + 600\,KN\cdot m$



7.8



$\Sigma M_B = 0$

$\Rightarrow (20\,KN)(2m) - (60\,KN)(6m) + D_y(8m) = 0$

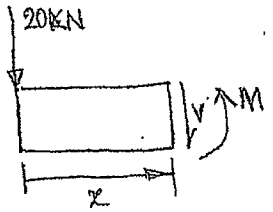
$\Rightarrow \quad D_y = 40\,KN$

$\Sigma f_y = 0$

$\Rightarrow \quad B_y + D_y - 20 - 60 = 0$

$\Rightarrow \quad B_y = 40\,KN$

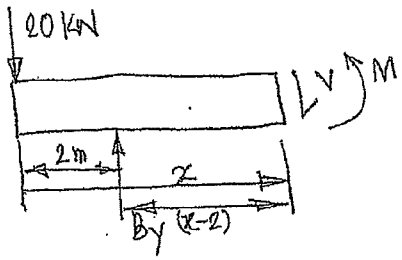
Section a-a' ($0 \le x \le 2m$)



$\Sigma f_y = -20KN - V = 0 \Rightarrow V = -20\,KN$

$\Sigma M_{a-a} = (20\,KN)\cdot x + M = 0 \Rightarrow M = -20x \ (KN\text{-}m)$

Section b-b' ($2m \le x \le 8m$)

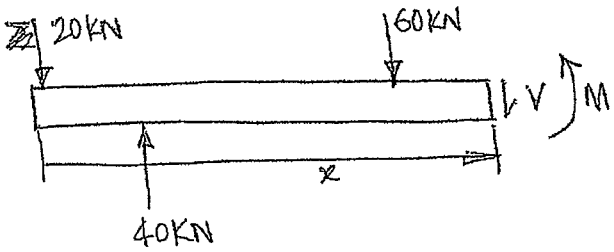


$\Sigma f_y = -20KN + B_y - V = 0 \Rightarrow V = 20\,KN$

$\Sigma M_{b-b} = 20x - B_y(x-2) + M = 0$

$\Rightarrow \quad M = 20x - 80$

Section c–c ($8m \le x \le 10m$):



$\Sigma F_y = -20 + B_y - 60 - V = 0$

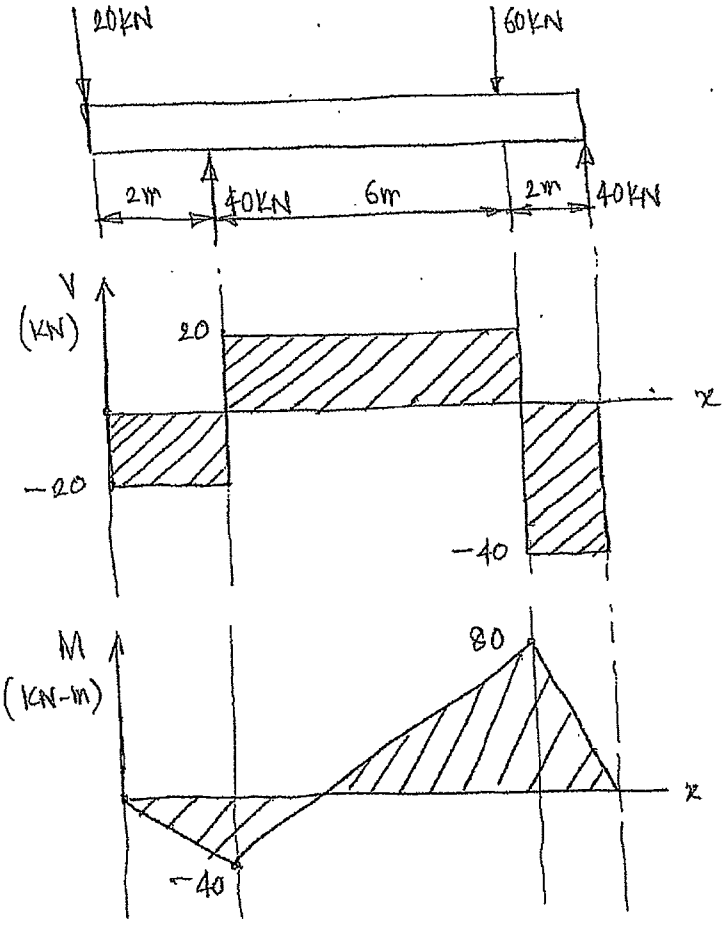
$\Rightarrow V = -40\ KN$

$\Sigma M_{cc} = 0$

$\Rightarrow 20x - 40(x-2) + 60(x-8) + M = 0$

$\Rightarrow M = -40x + 400$

(b) Shear-force and bendeing moment diagrams



(c) $M_{max-positive} = 80\ KN\text{-}m$ @ $x = 8m$

$M_{max-negative} = -40\ KN\text{-}m$ @ $x = 2m$