ME 325 – Machine design
Exam 1 – Feb. 23, 2009

1. Explain why the maximum stress that causes fatigue failure of a component is much lower than the maximum stress that causes failure of the same component under static loading. *(5 pts.)*

2. Explain briefly the fatigue life stages of a mechanical component subjected to fluctuating loads? *(5 pts.)*

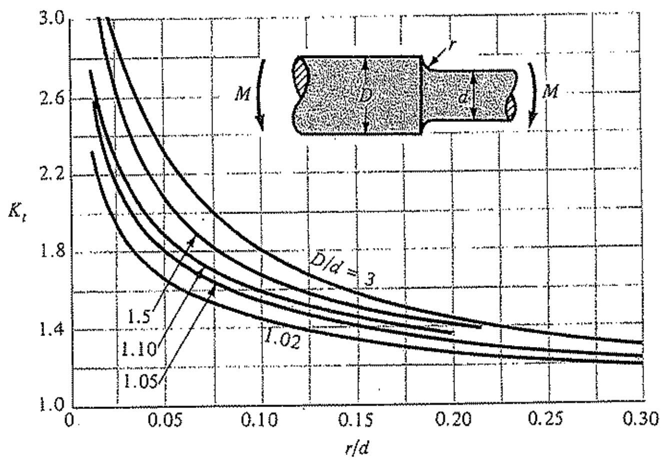
3. Explain why the rotating bending specimen is an important tool in the study of fatigue behavior of metals. *(5 pts.)*

4. Explain briefly what is the endurance limit. *(5 pts.)*

5. Calculate the fatigue strength of the rotating specimen shown in the figure below knowing that the fatigue life is $N = 15{,}000$ cycles. The specimen is machined of an AISI 4340 steel Q&T at 315°C. The dimensions are as follows: $d = 40$ mm, $D = 60$ mm, and $r = 6$ mm. *(40 pts.)*

FIGURE A-15-9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



6. A cylindrical pressure vessel with a 20-in. diameter and a 0.2-in wall thickness is made of a cold drawn steel with $S_{ut} = 95$ ksi, $S_y = 60$ ksi, and $\varepsilon_f = 0.15$. The pressure inside the cylinder varies between 0 and p_{max}.

(a) What is p_{max} that will cause static yielding?

(b) What is p_{max} for an infinite fatigue life of the cylinder?

(40 pts.)

(1) The maximum stress causing a fatigue failure is much lower than the maximum stress causing static failure because during fatigue loading it is sufficient to yield the material <u>only</u> locally near microscopic features. This stress is in general very low, much lower than the stress that will yield the entire cross-section of the specimen. The key concept here is local yielding versus global yielding.

(2) 1- <u>Crack nucleation</u> - the fatigue stage that starts at the beginning of loading cycles and ends when a crack becomes visible.

2 - <u>Crack growth</u> - it starts when the crack nucleation ends and it lasts until the last cycle of the total fatigue life. It is characterized by steady crack growth with each applied cycle.

3 - <u>Final failure</u> - is the last cycle when the specimen fails suddenly because the crack has grown long enough so that the remaining ligament cannot bear the applied stess. In general it has the features of a ductile fracture

(3) The rotating bending specimen is important in the study of fatigue because it allows the calculation of an "intrinsic" material property called endurance limit without considering the added influences of other factors such as: size, surface finish, loading type, etc.

(4) Endurance limit for a component loaded with fatigue loads that are completly reversed ($R=-1$) is the stress level that separates the finite life versus infinite life regions.

Applying a stress level bellow the endurance limit will not cause a fatigue failure of the component no matter how many cycles are applied. Applying a stress (which is alternating) above the endurance limit will cause the failure of a component after a finite number of cycles (for steels, in general less than 10^6 cycles)

5) $N = 15{,}000$ cycles

AISI 4340 steel Q & T 315°C $S_{ut} = 1720$ MPa

$$\left.\begin{array}{l} \frac{r}{d} = \frac{6}{40} = 0.15 \\ \frac{D}{d} = \frac{60}{40} = 1.5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} K_t = 1.5 \\ q = 0.95 \end{array}\right\} \Rightarrow \begin{array}{l} K_f = 1 + q\,(K_t - 1) \\ \quad = 1 + 0.95 \cdot 0.5 = 1.475 \end{array}$$

$$K_f' = a \cdot N^b \qquad a = \frac{1}{K_f} = \frac{1}{1.475} = 0.678$$

$$b = -\frac{1}{3} \log \frac{1}{K_f} = -\frac{1}{3} \log \frac{1}{1.475} = 0.056$$

$$K_f' = 0.678\,(15000)^{0.056} = 1.162$$

$$S_e' = 700 \text{ MPa} \qquad (S_{ut} > 1400 \text{ MPa})$$

$$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \cdot K_e \cdot S_e'$$

$$K_a = a \cdot S_{ut}^{\,b} = 4.51 \cdot (1720)^{-0.265} = 0.626$$

$$K_b = \left(\frac{d}{7.62}\right)^{-0.1133} = \left(\frac{40}{7.62}\right)^{-0.1133} = 0.829$$

$$K_c = 1 \qquad K_d = 1$$

$$K_e = \frac{1}{K_f'} = \frac{1}{1.162} = 0.861$$

$$S_e = K_a K_b K_c K_d K_e \cdot S_e' = 0.626 \cdot (0.829) \cdot (0.861) \cdot 700 = 313 \text{ MPa}$$

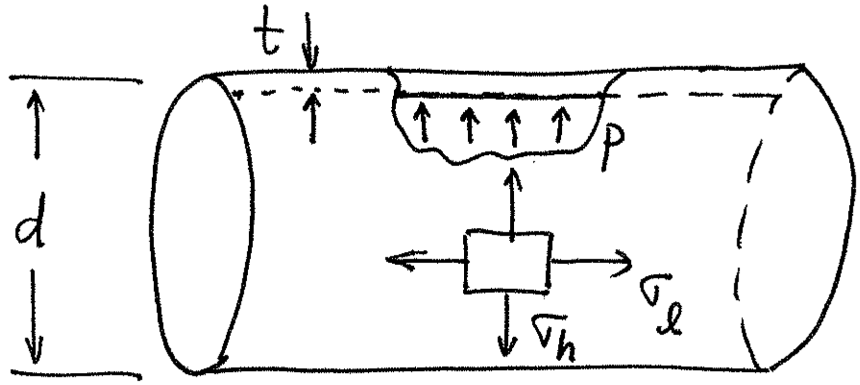
$$S_f = a \cdot N^b$$

$$a = \frac{(0.9\,S_{ut})^2}{S_e} = \frac{(0.9 \cdot 1720)^2}{313} = 7656 \quad \text{MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9\,S_{ut}}{S_e} = -\frac{1}{3}\log\frac{0.9 \cdot 1720}{313} = -0.231$$

$$S_f = 7656\,(15000)^{-0.235} = 799 \quad \text{MPa}$$

⑥ Cylindrical pressure vessel :



σ_l = longitudinal stress

σ_h = hoop stress

$$\sigma_l = \frac{pd}{4t} \qquad \sigma_h = \frac{pd}{2t}$$

$$\sigma_{l_{max}} = \frac{p_{max} \cdot 20}{4 \cdot 0.2} = 25\, p_{max}$$

$$\sigma_{h_{max}} = 2 \cdot \sigma_{l_{max}} = 50\, p_{max}$$

Von Mises stress

$$\sigma_{eq_{max}} = \sqrt{\sigma_{l_{max}}^2 + \sigma_{h_{max}}^2 - \sigma_{l_{max}} \sigma_{h_{max}}} =$$

$$= \sqrt{\sigma_{l_{max}}^2 + 4\sigma_{l_{max}}^2 - \sigma_{l_{max}} \cdot 2\sigma_{l_{max}}} =$$

$$= \sigma_{l_{max}} \sqrt{3} = 25\sqrt{3}\, p_{max} = 43.3\, p_{max}$$

(a) Assuming the material is ductile ($\varepsilon_f = 15\%$)

$$\sigma_{eq_{max}} = S_y \Rightarrow 43.3\, p_{max} = 60 \Rightarrow p_{max} = \frac{60}{43.3} = 1.38 \text{ Ksi}$$

(b) Goodman equation for infinite life:

$$\frac{\sigma_{eq\,a}}{S_e} + \frac{\sigma_{eq\,m}}{S_{ut}} = 1$$

$$\sigma_{eq_a} = \frac{\sigma_{eq\,max}}{2} = 21.65\ p_{max} \qquad (R=0)$$

$$\sigma_{eq_m} = \frac{\sigma_{eq\,max}}{2} = 21.65\ p_{max}$$

$$S_e = K_a K_b K_c K_d K_e \cdot (0.504\ S_{ut})$$

$$K_a = a \cdot (S_{ut})^b = 2.70 \cdot (95)^{-0.265} = 0.808$$

$K_b = 0.6$ (diameter larger than 2 in)

$K_c = 0.923$

$K_d = 1$

$K_e = 1$

$$S_e = 0.808 \cdot (0.6) \cdot (0.923)(0.504) \cdot 95 = 21.43\ \text{ksi}$$

Goodman equation:

$$\frac{21.65\ p_{max}}{21.43} + \frac{21.65\ p_{max}}{95} = 1 \quad \Rightarrow$$

$$\Rightarrow \quad p_{max} = \frac{1}{21.65} \cdot \left(\frac{1}{\frac{1}{21.43} + \frac{1}{95}} \right) = 0.808\ \text{ksi}$$