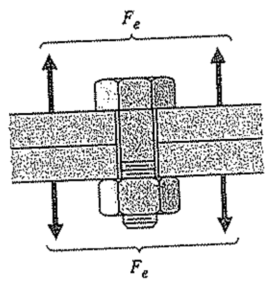
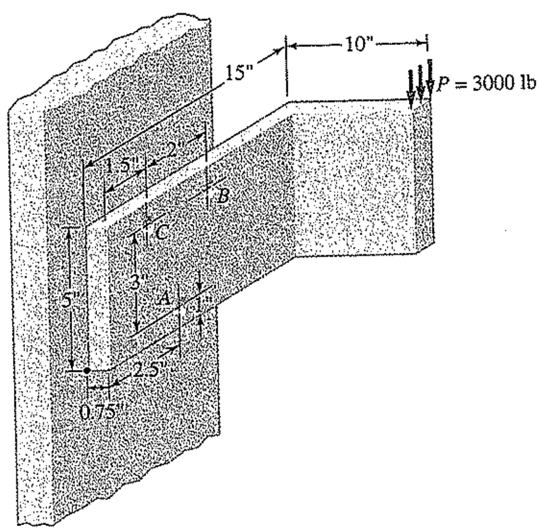
ME 325 Machine Design
Exam 2 – April 1, 2009

1. *(40 pts.)* The bolt shown below is *3/8" – 16 UNC SAE grade 5*, and it has cut threads. The bolt and the clamped plates are of the same length; the threads stop immediately above the nut. The clamped steel plates have a stiffness k_m six times the bolt stiffness k_b. The external load fluctuates between 0 and *8,000 lb*.
   (a) Find the minimum required value of initial preload to prevent loss of compression of the plates.
   (b) Find the minimum force in the plates when the preload is increased to *8,500 lb*.
   (c) Determine the number of cycles to failure for the bolt, if the connection is permanent.



2. *(60 pts.)* An L-shaped *1020* steel support bracket must support a static load of $P = 3000$ *lb*, as shown in the figure. A reusable connection with a bolt pattern using three bolts has been suggested. For each bolt, $k_m = 4k_b$. If the bolts are *1"- 8 UNC SAE grade 1*, determine the normal and the shear stresses in the bolts *A*, *B* and *C*.



① (a) $F_m = F_i + \frac{K_m}{K_m+K_b} P$

$F_m = 0 \Rightarrow F_i = -\frac{K_m}{K_m+K_b} P = -\frac{6K_b}{6K_b+K_b} \cdot 8000 = -6857 \text{ lb}$

(b) $F_i = -8500 \text{ lb}$

$F_m = F_i + \frac{K_m}{K_m+K_b} P_{max} = -8500 + \frac{6K_b}{6K_b+K_b} \cdot 8000 = -1643 \text{ lb}$

(c) Permanent connection

$F_i = 0.9\, S_p A_t \qquad S_p = 85 \text{ ksi} \qquad A_t = 0.0775 \text{ in}^2$

$F_i = 0.9 \cdot 85 \cdot 0.0775 = 5.929 \text{ kip} = 5929 \text{ lb}$

The force in the bolt

$F_b = F_i + CP$

$C = \frac{K_b}{K_m+K_b} = \frac{1}{7} = 0.143$

$F_{b_{max}} = F_i + C P_{max} \qquad F_{b_{min}} = F_i \Rightarrow$

$\Rightarrow F_{b_a} = \frac{F_{b\,max} - F_{b\,min}}{2} = \frac{C P_{max}}{2} = \frac{0.143 \cdot 8000}{2} = 572 \text{ lb}$

$F_{b_m} = \frac{F_{b\,max} + F_{b\,min}}{2} = F_i + C\frac{P_{max}}{2} = 5929 + 0.143 \cdot \frac{8000}{2} =$

$= 6501 \text{ lb}$

$$\sigma_a = \frac{F_{ba}}{A_t} = \frac{572}{0.0775} = 7.38 \text{ ksi}$$

$$\sigma_m = \frac{F_{bm}}{A_t} = \frac{6501}{0.0775} = 83.9 \text{ ksi}$$

Endurance limit for the bolt :

$S_e = 18.6$ ksi (Table 8-12)

Modified Goodman

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \Rightarrow n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{1}{\frac{7.38}{18.6} + \frac{83.9}{120}} = 0.912$$

$n < 1$ finite life

$$\frac{\sigma_a}{S_{N_f}} + \frac{\sigma_m}{S_{ut}} = 1 \Rightarrow S_{N_f} = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}} = \frac{7.38}{1 - \frac{83.9}{120}} = 24.53 \text{ ksi}$$

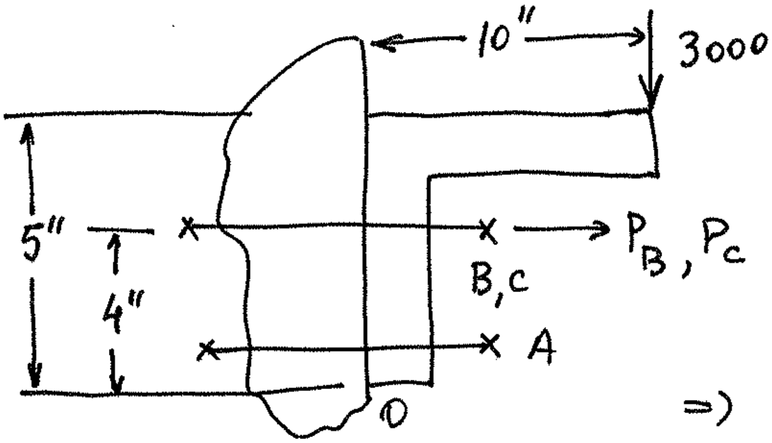
$$S_{N_f} = a \cdot N^b$$

$$a = \frac{(0.9\, S_{ut})^2}{S_e} = \frac{(0.9 \cdot 120)^2}{18.6} = 627.09 \text{ ksi}$$

$$b = -\frac{1}{3} \log \frac{0.9\, S_{ut}}{S_e} = -\frac{1}{3} \log \frac{0.9 \cdot 120}{18.6} = -0.255$$

$$N = \left(\frac{S_{N_f}}{a}\right)^{\frac{1}{b}} = \left(\frac{24.53}{627.09}\right)^{-\frac{1}{0.255}} = 313{,}227 \text{ cycles}$$

(2) First, calculate normal stresses in each bolt. Assume the external force on bolt A is zero.



$$\sum M_O = 0$$

$$\left.\begin{aligned}(P_B + P_C)\,4'' &= 3000 \cdot 10'' \\ P_B &= P_C\end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \quad P_B = P_C = P \cdot \frac{10}{8} = 3{,}000 \cdot \frac{10}{8} = 3750 \text{ lb}$$

$$F_i = 0.75 \cdot S_p \cdot A_t = 0.75 \cdot 33 \cdot 0.606 = 15 \text{ Kip}$$

$S_p = 33$ ksi (Table 8-4)

$A_t = 0.606$ in^2 (Table 8-2)

For each bolt $C = \dfrac{K_b}{K_m + K_b} = \dfrac{1}{5} = 0.2$

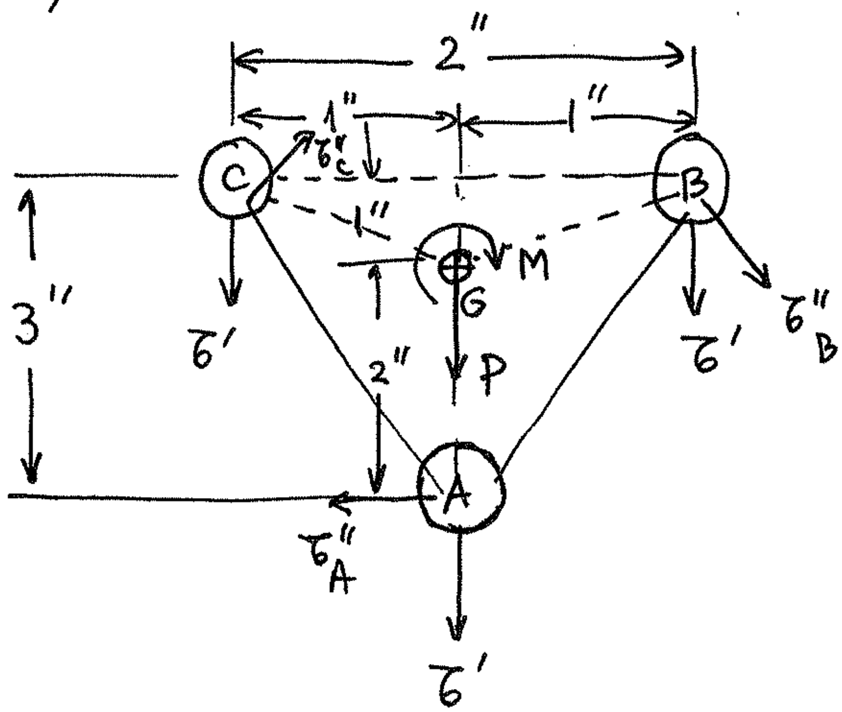
$$F_{b_B} = F_i + C\,P_B = 15{,}000 + 0.2 \cdot 3750 = 15{,}750 \text{ lb}$$

$$F_{b_C} = F_{b_B} = 15{,}750 \text{ lb}$$

$$\sigma_B = \sigma_C = \frac{F_{b_B}}{A_t} = \frac{15{,}750}{0.606} = 25.9 \text{ Ksi}$$

$$\sigma_A = \frac{F_i}{A_t} = \frac{15{,}000}{0.606} = 24.7 \text{ Ksi}$$

Second, let's calculate the shear stresses in the bolts A, B and C.



$P = 3000$ lb

$M = P \cdot (15'' - 1.5'' - 1'') =$

$= P \cdot 12.5 =$

$= 3000 \cdot (12.5) =$

$= 37.5 \cdot 10^3$ lb·in

$$\tau' = \frac{P}{3A_t} = \frac{3000}{3 \cdot 0.606} = 1.65 \text{ ksi}$$

$r_B = GB = \sqrt{1+1} = \sqrt{2} = 1.414$ in

$r_C = GC = 1.414$ in

$r_A = 2$ in

Secondary shear stresses:

$r_A^2 + r_B^2 + r_C^2 = 2 + 2 + 4 = 8$ in^2

$$F_A = \frac{M r_A}{r_A^2 + r_B^2 + r_C^2} = \frac{37.5 \cdot 10^3 \cdot 2}{8} = 9375 \text{ lb}$$

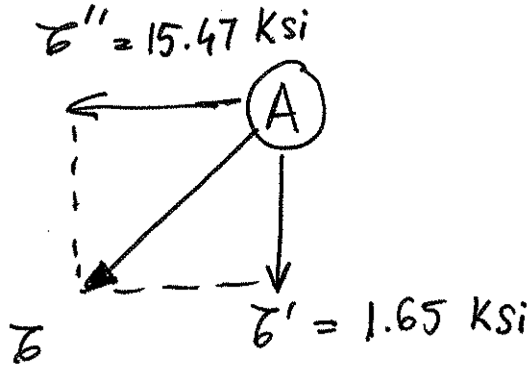
$$F_B = F_C = \frac{M r_B}{r_A^2 + r_B^2 + r_C^2} = \frac{37.5 \cdot 10^3 \cdot \sqrt{2}}{8} = 6629 \text{ lb}$$

$$\tau_A'' = \frac{F_A}{A_t} = \frac{9375}{0.606} = 15.47 \text{ Ksi}$$

$$\tau_B'' = \frac{F_B}{A_t} = \frac{6629}{0.606} = 10.94 \text{ Ksi}$$

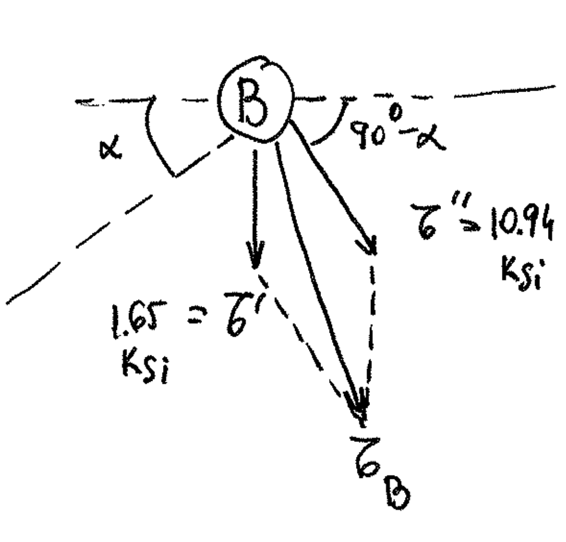
$$\tau_C'' = \tau_B'' = 10.94 \text{ Ksi}$$

Total shear stresses:



$$\tau_A = \sqrt{\tau'^2 + \tau''^2} = \sqrt{1.65^2 + 15.47^2}$$

$$= 15.56 \text{ Ksi}$$



$\alpha = 45^\circ$

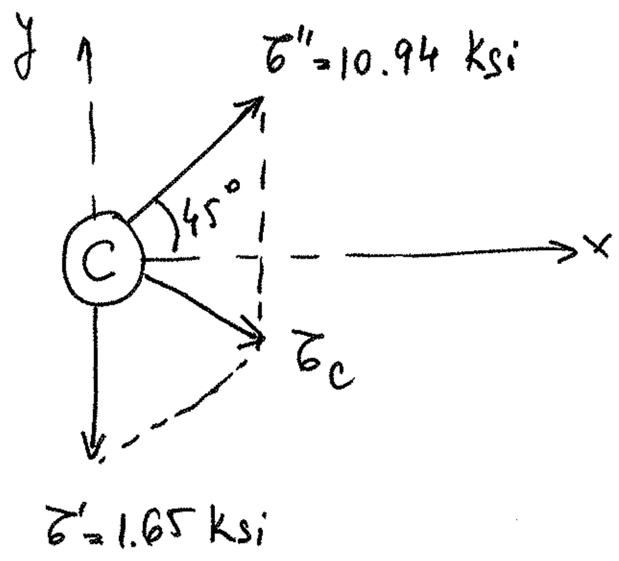
$90^\circ - \alpha = 45^\circ$

$$\tau_x'' = \tau'' \cdot \cos 45^\circ = 10.94 \cdot \frac{\sqrt{2}}{2} = 7.74 \text{ Ksi}$$

$$\tau_y'' = \tau'' \cdot \sin 45^\circ = 7.74 \text{ Ksi}$$

$$\tau_B = \sqrt{7.74^2 + (7.74 + 1.65)^2} =$$

$$= 12.17 \text{ Ksi}$$



$$\tau''_x = \tau'' \cos 45° = 7.74 \text{ ksi}$$

$$\tau''_y = \tau'' \sin 45° = 7.74 \text{ ksi}$$

$$\tau_C = \sqrt{\tau''^2_x + (\tau''_y - \tau')^2} = \sqrt{7.74^2 + (7.74 - 1.65)^2} =$$

$$= 9.85 \text{ ksi}$$

The largest shear stress at A $\tau_A = 15.56$ ksi