1. (40 pts.) The bolt shown below is 3/8" - 16 UNC SAE grade 5, and it has cut threads. The bolt and the clamped plates are of the same length; the threads stop immediately above the nut. The clamped steel plates have a stiffness $k_m$ six times the bolt stiffness $k_b$. The external load fluctuates between 0 and $8,000 \text{ lb}$.

(a) Find the minimum required value of initial preload to prevent loss of compression of the plates.

(b) Find the minimum force in the plates when the preload is increased to $8,500 \text{ lb}$.

(c) Determine the number of cycles to failure for the bolt, if the connection is permanent.

2. (60 pts.) An L-shaped 1020 steel support bracket must support a static load of $P = 3000 \text{ lb}$, as shown in the figure. A reusable connection with a bolt pattern using three bolts has been suggested. For each bolt, $k_m = 4k_b$. If the bolts are 1" - 8 UNC SAE grade 1, determine the normal and the shear stresses in the bolts $A$, $B$ and $C$. 

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**Diagram:**

- Bolt with threads.
- Clamped plates with stiffnesses.
- External load indicated.

- L-shaped bracket with load $P = 3000 \text{ lb}$.
- Bolt pattern with bolts $A$, $B$, and $C$.

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1. (a) \[ F_m = F_i + \frac{Km}{K_m + K_b} \cdot P \]

\[ F_p = 0 \rightarrow F_i = -\frac{Km}{K_m + K_b} \cdot P = -\frac{6K_b}{6K_b + K_b} \cdot 8000 = -6857 \text{ lb} \]

(b) \[ F_i = -8500 \text{ lb} \]

\[ F_m = F_i + \frac{Km}{K_m + K_b} \cdot P_{\text{max}} = -8500 + \frac{6K_b}{6K_b + K_b} \cdot 8000 = -1643 \text{ lb} \]

(c) Permanent connection

\[ F_i = 0.9 \cdot S_p A_t \]

\[ S_p = 85 \text{ ksi} \quad A_t = 0.0775 \text{ in}^2 \]

\[ F_i = 0.9 \cdot 85 \cdot 0.0775 = 5.929 \text{ kip} = 5929 \text{ lb} \]

The force in the bolt

\[ F_b = F_i + C \cdot P \]

\[ C = \frac{K_b}{K_m + K_b} = \frac{1}{7} = 0.143 \]

\[ F_{b,\text{max}} = F_i + C \cdot P_{\text{max}} \quad F_{b,\text{min}} = F_i \quad \rightarrow \]

\[ \Rightarrow F_{ba} = \frac{F_{b,\text{max}} - F_{b,\text{min}}}{2} = \frac{C \cdot P_{\text{max}}}{2} = \frac{0.143 \cdot 8000}{2} = 572 \text{ lb} \]

\[ F_{b} = \frac{F_{b,\text{max}} + F_{b,\text{min}}}{2} = F_i + C \cdot \frac{P_{\text{max}}}{2} = 5929 + 0.143 \cdot \frac{8000}{2} = 6501 \text{ lb} \]
\[ \sigma_a = \frac{F_{ba}}{A_t} = \frac{572}{0.0775} = 7.38 \text{ ksi} \]

\[ \sigma_m = \frac{F_{bm}}{A_t} = \frac{6501}{0.0775} = 83.9 \text{ ksi} \]

*Endurance limit for the bolt:*

\[ S_e = 18.6 \text{ ksi} \quad (\text{Table 8-12}) \]

*Modified Goodman*

\[ \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} > \frac{1}{N} \quad \Rightarrow \quad N = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{1}{\frac{7.38}{18.6} + \frac{83.9}{120}} = 0.912 \]

\[ N < 1 \quad \text{finite life} \]

\[ \frac{\sigma_a}{S_{Nf}} + \frac{\sigma_m}{S_{ut}} = 1 \quad \Rightarrow \quad S_{Nf} = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}} = \frac{7.38}{1 - \frac{83.9}{120}} = 24.53 \text{ ksi} \]

\[ S_{Nf} = a \cdot N^b \]

\[ a = \frac{(0.9 \cdot S_{ut})^2}{S_e} = \frac{(0.9 \cdot 120)^2}{18.6} = 627.09 \text{ ksi} \]

\[ b = -\frac{1}{3} \log \frac{0.9 \cdot S_{ut}}{S_e} = -\frac{1}{3} \log \frac{0.9 \cdot 120}{18.6} = -0.255 \]

\[ N = \left( \frac{S_{Nf}}{a} \right)^\frac{1}{b} = \left( \frac{24.53}{627.09} \right)^{-0.255} = 313,227 \text{ cycles} \]
First, calculate normal stresses in each bolt. Assume the external force on bolt A is zero.

\[ 3000 \text{ lb} = P \quad \Sigma M_0 = 0 \]
\[ (P_B + P_C) \times 4" = 3000 \times 10" \]
\[ P_B = P_C \]
\[ P_B = P_C = P \cdot \frac{10}{8} = 3000 \cdot \frac{10}{8} = 3750 \text{ lb} \]

\[ F_i = 0.75 \cdot S_p \cdot A_t = 0.75 \cdot 33 \cdot 0.606 = 15 \text{ Kip} \]

\[ S_p = 33 \text{ ksi (Table 8-4)} \]

\[ A_t = 0.606 \text{ in}^2 \text{ (Table 8-2)} \]

For each bolt: \( C = \frac{K_b}{K_m + K_b} \sim \frac{1}{5} = 0.2 \)

\[ F_{b_B} = F_i + C \cdot P_B = 15,000 + 0.2 \cdot 3750 = 15,750 \text{ lb} \]

\[ F_{b_C} = F_{b_B} = 15,750 \text{ lb} \]

\[ \sigma_B = \frac{F_{b_B}}{A_t} = \frac{15,750}{0.606} = 25.9 \text{ Ksi} \]

\[ \sigma_A = \frac{F_i}{A_t} = \frac{15,000}{0.606} = 24.7 \text{ Ksi} \]
Second, let's calculate the shear stresses in the bolts A, B, and C.

\[ P = 3000 \text{ lb} \]
\[ M = P \cdot (15'' - 1.5'' - 1'') = P \cdot 12.5 = 3000 \cdot (12.5) = 37500 \text{ lb} \cdot \text{in} \]

\[ \tau' = \frac{P}{3A_t} = \frac{3000}{3 \cdot 0.606} = 1.65 \text{ ksi} \]

\[ r_B = GB = \sqrt{1 + 1} = \sqrt{2} = 1.414 \text{ in} \]
\[ r_C = GC = 1.414 \text{ in} \]
\[ r_A = 2 \text{ in} \]

Secondary shear stresses:
\[ r_A^2 + r_B^2 + r_C^2 = 2^2 + 2^2 + 4^2 = 8 \text{ in}^2 \]
\[ F_A = \frac{M \, r_A}{r_A^2 + r_B^2 + r_C^2} = \frac{37.5 \cdot 10^3 \cdot 2}{8} = 9375 \text{ lb} \]

\[ F_B = F_c = \frac{M \, r_B}{r_A^2 + r_B^2 + r_C^2} = \frac{37.5 \cdot 10^3 \cdot \sqrt{2}}{8} = 6629 \text{ lb} \]

\[ \varepsilon''_A = \frac{F_A}{A_t} = \frac{9375}{0.606} = 15.47 \text{ KSI} \]

\[ \varepsilon''_B = \frac{F_B}{A_t} = \frac{6629}{0.606} = 10.94 \text{ KSI} \]

\[ \varepsilon''_C = \varepsilon''_B = 10.94 \text{ KSI} \]

**Total shear stresses:**

\[ \varepsilon'' = 15.47 \text{ KSI} \]

\[ \varepsilon = \sqrt{\varepsilon''^2 + \varepsilon''^2} = \sqrt{1.65^2 + 15.47^2} \]

\[ = 15.56 \text{ KSI} \]

\[ \alpha = 45^\circ \]

\[ \alpha = 45^\circ \]

\[ 90^\circ - \alpha = 45^\circ \]

\[ \varepsilon''_x = \varepsilon'' \cdot \cos 45^\circ = 10.94 \cdot \frac{\sqrt{2}}{2} = 7.74 \text{ KSI} \]

\[ \varepsilon''_y = \varepsilon'' \cdot \sin 45^\circ = 7.74 \text{ KSI} \]

\[ \varepsilon_B = \sqrt{7.74^2 + (7.74 + 1.65)^2} \]

\[ = 12.17 \text{ KSI} \]
\[ \sigma_x = 6" \cos 45^\circ = 7.74 \text{ KSI} \]
\[ \sigma_y = 6" \sin 45^\circ = 7.74 \text{ KSI} \]
\[ \sigma_C = \sqrt{\sigma_x^2 + (\sigma_y - \sigma')^2} = \sqrt{7.74^2 + (7.74 - 1.65)^2} = \]
\[ = 9.85 \text{ KSI} \]

The largest shear stress at A \[ \tau_A = 15.56 \text{ KSI} \]