ME 325 Machine Design
Final Exam, May 12, 2009

1. (*4 pts.*) What are the three stages of a fatigue failure?

- crack nucleation
- crack growth
- final failure

2. (*4 pts.*) What is fatigue endurance limit for a metallic component?

The fatigue endurance limit is the alternating stress below which any applied alternating stress will not cause a fatigue failure.

3. (*4 pts.*) What is proof strength for a bolt?

Proof strength is the proof load divided by bolt area. Proof load is the maximum applied load that will not cause a permanent deformation in the bolt.

4. (*4 pts.*) A bolt is clamping two plates. During preload tightening the stresses induced in the bolt and clamping members are compressive. True or false?

False. The bolt is loaded in tension and the plates in compression.

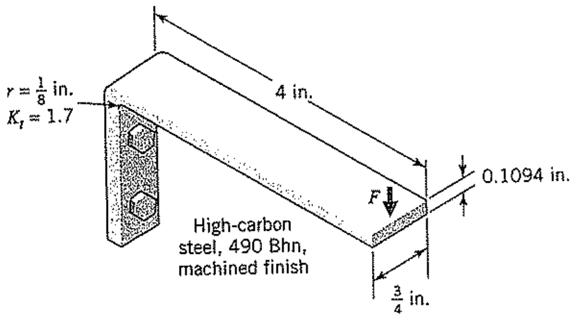
5. (*4 pts.*) What are the three main steps in solving a finite element problem?

1. Pre-processing
2. Solution
3. Post-processing

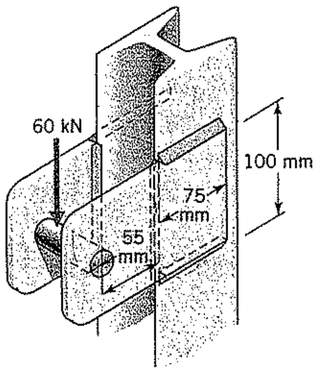
6. (*4 pts.*) A helical spring is loaded with a compressive force. The stresses resulting in the spring wire are compressive normal stresses. True or false?

False. The spring wire will experience shear stresses.

7. (*25 pts.*) The figure shows a cantilever beam serving as a spring for a latching mechanism. When assembled, the free end is deflected 0.075 in., which corresponds to a force F of *8.65* lb. When a latch operates, the end deflects an additional 0.15 in. Would you expect eventual fatigue failure? Use S_{ut} (Ksi) = 0.5·Bhn.

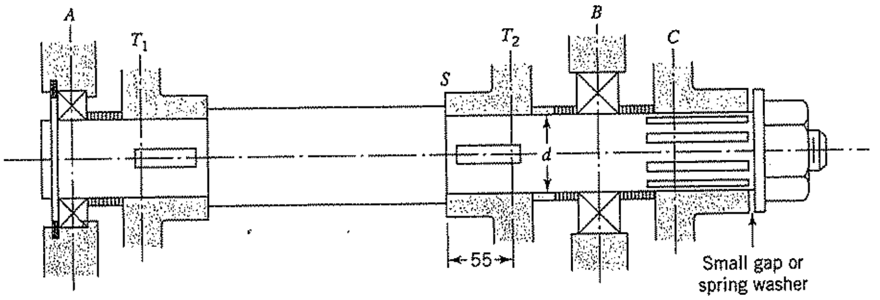


8. (*25 pts*) The bracket shown in the figure supports a total load of *60 kN* equally divided between the two sides. Using a factor of safety of *3.0* and knowing that the yield strength of the weld material is $S_y = 325\ MPa$, what size weld should be used?

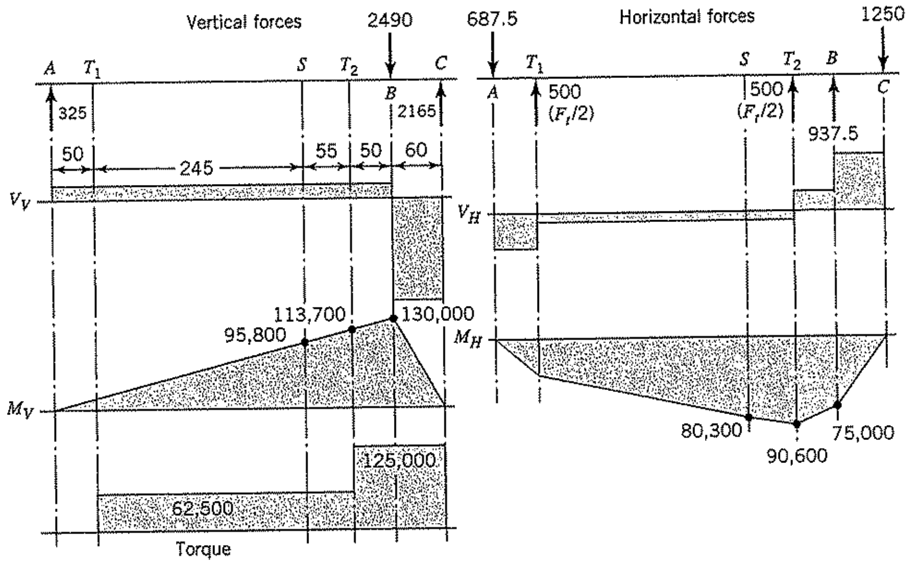


Note: Each plate has two 75 mm welds and one 100 mm weld.

9. (*25 pts.*) A snowmobile drive shaft is supported in the frame by bearings A and B, and is chain-driven by sprocket C. The loading on the shaft creates the moment and force diagrams (in N and mm) as shown. Considering the maximum distortion energy criterion, determine the required diameter d between points S and B, given a factor of safety of 3.0 against monotonic failure. The material yield strength is $400\ MPa$.



(*b*) Shaft layout



(*c*) Loading diagrams

(7) $K = \dfrac{F}{\delta}$

$F_1 = 8.65\ lb \quad \delta_1 = 0.075\ in$

$F_2 = ? \quad \delta_2 = 0.075 + 0.15 = 0.225\ in$

$$\frac{F_1}{\delta_1} = \frac{F_2}{\delta_2} \Rightarrow F_2 = F_1 \cdot \frac{\delta_2}{\delta_1} = 8.65 \cdot \frac{0.225}{0.075} = 25.95\ lb$$

During one cycle:

$F_{min} = 8.65\ lb \quad F_{max} = 25.95\ lb$

$M_{min} = F_{min} \cdot 4 = 8.65 \cdot 4 = 34.6\ lb \cdot in$

$M_{max} = F_{max} \cdot 4 = 25.95 \cdot 4 = 103.8\ lb \cdot in$

$$\sigma_{min} = \frac{M_{min} \cdot c}{I} = \frac{34.6 \cdot \frac{0.1094}{2}}{\frac{0.75 \cdot (0.1094)^3}{12}} = 23.13\ Ksi$$

$$\sigma_{max} = \frac{M_{max} \cdot c}{I} = \frac{103.8 \cdot \frac{0.1094}{2}}{\frac{0.75 \cdot (0.1094)^3}{12}} = 69.38\ ksi$$

$S_{ut} = 0.5 \cdot Bhn = 0.5 \cdot 490 = 245\ Ksi$

$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \cdot K_e \cdot (0.504\ S_{ut})$

$K_a = a \cdot S_{ut}^{\,b} = 2.7 \cdot (245)^{-0.265} = 0.628$

$$K_b = \left(\frac{d_e}{0.3}\right)^{-0.1133} = \left(\frac{0.231}{0.3}\right)^{-0.1133} = 1.03$$

$$d_e = 0.808\sqrt{hb} = 0.808\sqrt{0.75 \cdot 0.1094} = 0.231 \text{ in}$$

$$K_c = 1 \quad (\text{bending})$$

$$K_d = 1$$

$$K_e = \frac{1}{K_f} = \frac{1}{1.67} = 0.6$$

$$K_f = 1 + q(K_t - 1) = 1 + 0.95(1.7-1) = 1.67$$

$$\left.\begin{array}{l} r = \frac{1}{8} = 0.125 \text{ in} \\ S_{ut} = 245 \text{ Ksi} \end{array}\right\} \Rightarrow q \cong 0.95$$

$$S_e' = 100 \text{ Ksi} \quad (S_{ut} > 200 \text{ Ksi})$$

$$S_e = 0.628 \cdot 1.03 \cdot 0.6 \cdot 100 = 38.81 \text{ Ksi}$$

Modified Goodman

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \Rightarrow n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

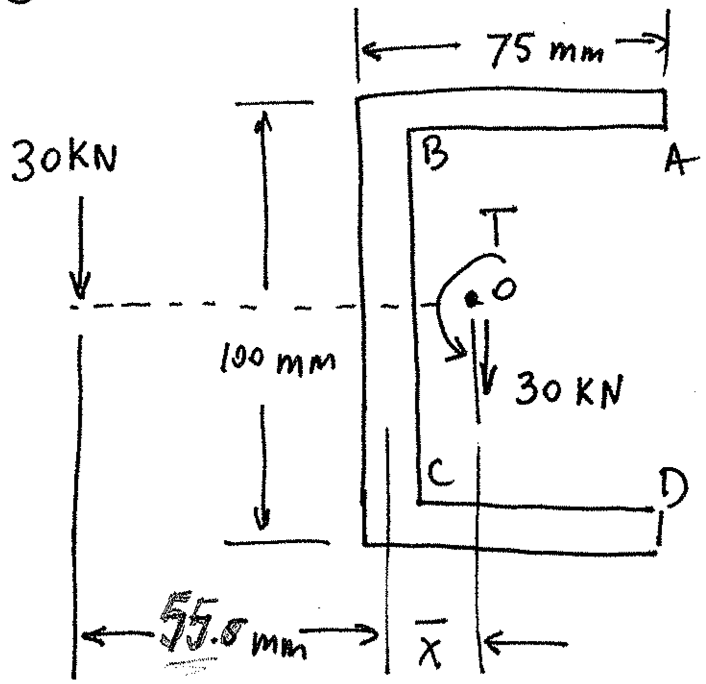
$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{69.38 - 23.13}{2} = 23.12 \text{ Ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{69.38 + 23.13}{2} = 46.25 \text{ Ksi}$$

$$n = \frac{1}{\frac{23.12}{38.81} + \frac{46.25}{245}} = 1.27$$

safe; the spring will not fail due to fatigue loading.

(8)



$$\bar{x} = \frac{75^2}{2\cdot 75+100} = 22.5 \text{ mm}$$

$$A_u = 2b+d = 250 \text{ mm}$$

$$J_u = \frac{8\cdot 75^3 + 6\cdot 75\cdot 100^2 + 100^3}{12} - \frac{75^4}{2\cdot 75+100} = 6.13\cdot 10^5 \text{ mm}^3$$

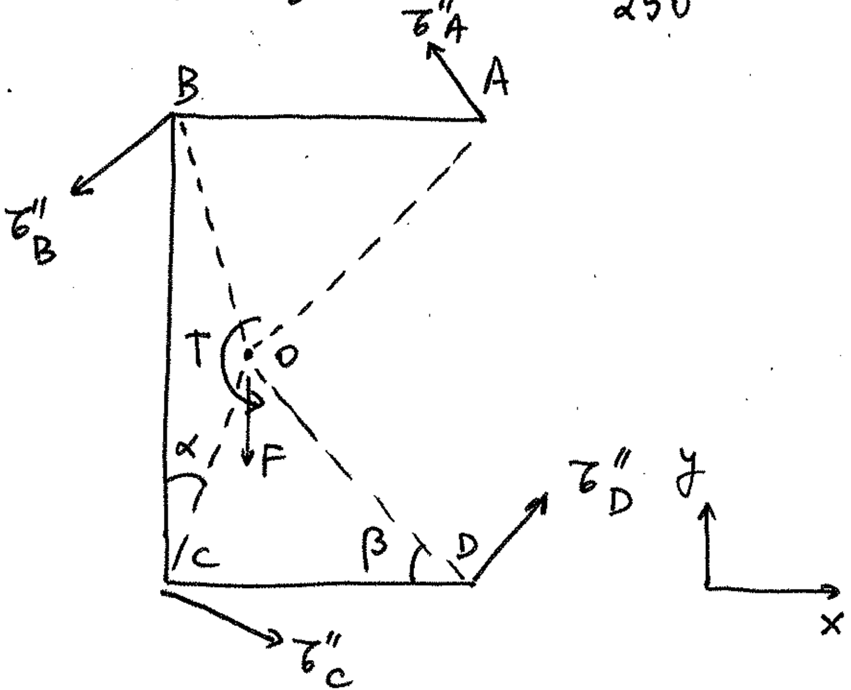
$F = 30$ KN $\quad T = F\cdot 77.5 = 30\cdot 77.5 = 2325$ KN·mm

We are going to solve this problem in terms of $t = 0.707\,h$

$$OB = OC = \sqrt{22.5^2 + 50^2} = 54.83 \text{ mm}$$

$$OA = OD = \sqrt{52.5^2 + 50^2} = 72.5 \text{ mm}$$

$$\tau'_A = \tau'_B = \tau'_C = \tau'_D = \frac{30\cdot 10^3}{250} = 120 \ \frac{N}{mm}$$ (stress per unit thickness)



$$\sin\alpha = \frac{22.5}{54.83} = 0.41$$

$$\cos\alpha = \frac{50}{54.83} = 0.912$$

$$\sin\beta = \frac{50}{72.5} = 0.69$$

$$\cos\beta = \frac{52.5}{72.5} = 0.724$$

$$\tau''_{A_x} = \tau''_A \cdot \sin\beta = 275 \cdot 0.69 = 189.75 \ \frac{N}{mm}$$

$$\tau''_{A_y} = \tau''_A \cdot \cos\beta = 275 \cdot 0.724 = 199 \ \frac{N}{mm}$$

$$\tau''_A = \frac{T \cdot r_A}{J_u} = \frac{2325 \cdot 72.5}{6.13 \cdot 10^5} = 0.275 \ \frac{KN}{mm} = 275 \ \frac{N}{mm}$$

$$\tau''_D = \tau''_A = 275 \ \frac{N}{mm}$$

$$\tau''_{D_x} = \tau''_D \cdot \sin\beta = 189.75 \ \frac{N}{mm}$$

$$\tau''_{D_y} = \tau''_D \cdot \cos\beta = 199 \ \frac{N}{mm}$$

$$\tau''_C = \tau''_D = T \cdot \frac{r_C}{J} = \frac{2325 \cdot 54.83}{6.13 \cdot 10^5} = 208 \ \frac{N}{mm}$$

$$\tau''_{C_x} = \tau''_C \cdot \cos\alpha = 208 \cdot 0.912 = 189.7 \ \frac{N}{mm}$$

$$\tau''_{C_y} = \tau''_C \cdot \sin\alpha = 208 \cdot 0.41 = 85.3 \ \frac{N}{mm}$$

The largest shear stresses occur at B and C :

$$\tau_C = \sqrt{\tau''^{2}_{Cx} + \left(\tau''_{Cy} + \tau'_C\right)^2} = \sqrt{189.7^2 + \left(85.3 + 120\right)^2} = 279 \ \frac{N}{mm}$$

The shear stress is $\tau_C = \frac{280}{t} = \frac{280}{0.707h}$

$$n = \frac{S_{sy}}{\tau_C} = \frac{0.5\, S_y}{\tau_C} = \frac{0.5\, S_y}{\frac{280}{0.707h}} = \frac{0.5 \cdot 0.707h \cdot S_y}{280} \Rightarrow$$

$$\Rightarrow \quad h = \frac{280 \cdot n}{0.5 \cdot 0.707 \cdot S_y} = \frac{280 \cdot 3}{0.5 \cdot 0.707 \cdot 325} = 7.31 \text{ mm}$$

(9) Maximum distortion energy criterion :

$$d = \sqrt[3]{\frac{16\,n}{\pi S_y}\sqrt{4M^2+3T^2}}$$

The total bending moments at points S, T_2 and B are :

$$M_S = \sqrt{M_V^2 + M_H^2} = \sqrt{95{,}800^2 + 80{,}300^2} = 125{,}000 \ \text{N}\cdot\text{mm}$$

$$M_{T_2} = \sqrt{113{,}700^2 + 90{,}600^2} = 145{,}382 \ \text{N}\cdot\text{m}$$

$$M_B = \sqrt{130{,}000^2 + 75{,}000^2} = 150{,}083 \ \text{N}\cdot\text{m}$$

The largest moment is at B, this is where we will calculate d :

$$d = \sqrt[3]{\frac{16\cdot 3.0}{\pi\cdot 400}\sqrt{4\cdot 150{,}083^2 + 3\cdot 125{,}000^2}} = 24.18 \ \text{mm}$$