ME 325 Machine Design
Final Exam, May 15, 2009

1. *(5 pts.)* Why correction factors are introduced when calculating the stress in a helical spring?

2. *(5 pts.)* A bolt is clamping two plates together. List all the possible failure modes (loading modes) for the bolt.

3. *(5 pts.)* Rough surfaces lead to improved fatigue lives for metallic components. True of false? Explain.

4. *(5 pts.)* Which is the strongest spring wire material commonly used for helical springs?

5. *(5 pts.)* What is the most common failure mode for roller bearings?

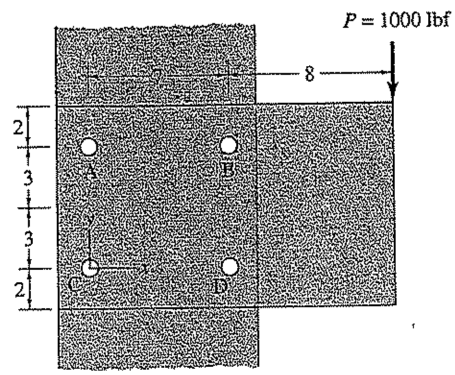
6. *(25 pts.)* The member shown in the figure is fastened to a beam by means of four bolts, and an eccentric load is applied. Bolts *A* and *C* are 5/8-in. in diameter and bolts *B* and *D* are 7/8-in. in diameter. Determine the following:

a) centroid of the bolt assembly;

b) the total shear stress on each of the bolts;

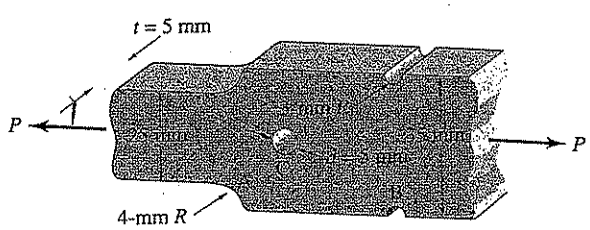
c) the safety factor guarding against shear of the bolt.

Assume that the yield stress of the bolt material is 85 ksi.

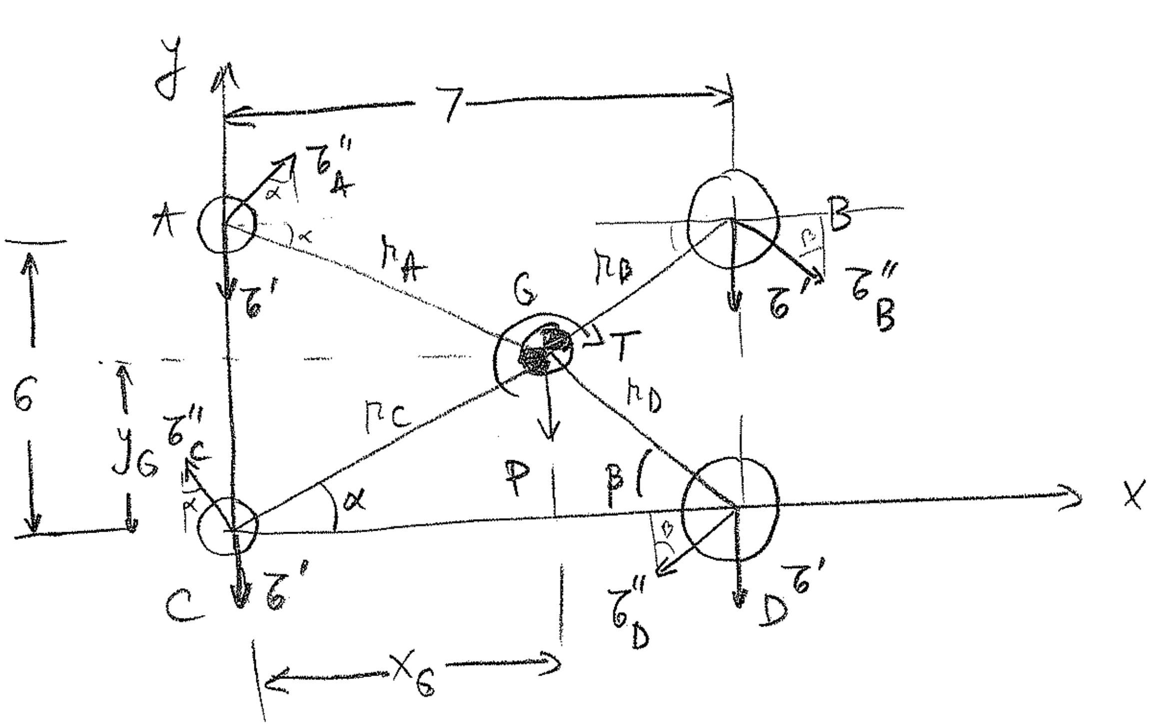


7. (25 pts.) A compression coil spring is made of music wire with squared and ground ends. The spring is to have a spring rate of *1250 N/m*. The force corresponding to a solid length is *60 N*. At the solid length the shear stress in the wire reaches its corresponding yield limit. The spring index is *10*. Find the wire diameter and the mean coil diameter.

8. The bar shown below is made of hot-rolled *AISI 1080*. The applied load fluctuates between a minimum of *2 kN* and a maximum of *10 kN*. Using the modified Goodman failure theory determine the factors of safety for the hole, the fillet and the groove.



⑥ a)



$$d_A = d_C = \frac{5}{8}'' \Rightarrow A_A = A_C = \frac{\pi d^2}{4} = \frac{\pi \cdot \left(\frac{5}{8}\right)^2}{4} = 0.307 \text{ in}^2$$

$$d_B = d_D = \frac{7}{8}'' \Rightarrow A_B = A_D = \frac{\pi d^2}{4} = \frac{\pi \left(\frac{7}{8}\right)^2}{4} = 0.601 \text{ in}^2$$

$$X_G = \frac{A_A x_A + A_C \cdot x_C + A_B \cdot x_B + A_D \cdot x_D}{A_A + A_C + A_B + A_D} = \frac{0.601 \cdot 7 + 0.601 \cdot 7}{2 \cdot 0.307 + 2 \cdot 0.601} = 4.66 \text{ in}$$

$y_G = 3$ (by symmetry)

$$r_C = r_A = \sqrt{x_G^2 + y_G^2} = \sqrt{4.66^2 + 3^2} = 5.54 \text{ in}$$

$$r_D = r_B = \sqrt{(7 - x_G)^2 + y_G^2} = \sqrt{(7 - 4.66)^2 + 3^2} = 3.805 \text{ in}$$

$$\sin\alpha = \frac{3}{5.54} = 0.542 \qquad \sin\beta = \frac{3}{3.805} = 0.788$$

$$\cos\alpha = \frac{4.66}{5.54} = 0.841 \qquad \cos\beta = \frac{7 - 4.66}{3.805} = 0.615$$

$$\tau' = \frac{P}{\sum A_i} = \frac{1000}{2\cdot 0{,}307 + 2\cdot 0{,}601} = 551 \text{ psi}$$

$$T = P\cdot d = 1000\cdot(8+7-4{,}66) = 10340 \text{ lb}\cdot\text{in}$$

$$F_A'' = \tau_A''\cdot A_A = \frac{T\cdot r_A}{r_A^2 + r_B^2 + r_C^2 + r_D^2} = \frac{10340\cdot 5{,}54}{2\cdot 5{,}54^2 + 2\cdot 3{,}805^2} = 634 \text{ lb}$$

$$\tau_A'' = \tau_C'' = \frac{634 \text{ lb}}{0{,}307 \text{ in}^2} = 2065 \text{ psi}$$

$$F_B'' = \tau_B''\cdot A_B = \frac{T\cdot r_B}{r_A^2 + r_B^2 + r_C^2 + r_D^2} = \frac{10340\cdot 3{,}805}{2\cdot 5{,}54^2 + 2\cdot 3{,}805^2} = 436 \text{ lb}$$

$$\tau_B'' = \frac{436}{0{,}601} = 725 \text{ psi}$$

Points A and C

$$\tau_A = \sqrt{(\tau_A'' \sin\alpha)^2 + (\tau_A'' \cos\alpha - \tau')^2} = \sqrt{(2065\cdot 0{,}542)^2 + (2065\cdot 0{,}841 - 551)^2}$$

$$= 1630 \text{ psi}$$

$$\tau_C = \tau_A = 1630 \text{ psi}$$

Points B and D

$$\tau_B = \sqrt{(\tau_B''\cdot \sin\beta)^2 + (\tau_B''\cdot \cos\beta - \tau')^2} = \sqrt{(725\cdot 0{,}788)^2 + (725\cdot 0{,}615 - 551)^2} =$$

$$= 1149 \text{ psi}$$

$$\tau_B = \tau_D = 1149 \text{ psi}$$

Maximum allowable shear stress :

$$\tau_{all} = 0.5 \cdot S_y = 0.5 \cdot 85 = 42.5 \text{ Ksi}$$

$$n = \frac{\tau_{all}}{\tau_D} = \frac{42.5}{1.63} = 26$$

<u>Note</u> : The solution is not exact. For the secondary shear stress we made the assumption that all the bolts have the same area, because we did not solve an example when the bolts have different areas. The secondary shear stress should be calulated with:

$$\tau_i'' = \frac{T r_i}{\sum_{j=1}^{4} r_j^2 A_j}$$

For example

$$\tau_A'' = \frac{T \cdot r_A}{r_A^2 \cdot A_A + r_B^2 A_B + r_C^2 \cdot A_C + r_D^2 A_D}$$

(7) Music wire:

$$S_{ut} = \frac{A}{d^m}$$

Assuming that yielding occurs at the solid length

$$\tau = K_s \cdot \frac{8F \cdot D}{\pi d^3} = K_s \cdot \frac{8F \cdot D}{\pi d^2 \cdot d} = K_s \frac{8FC}{\pi d^2}$$

$$K_s = \frac{2C+1}{2C} = \frac{2 \cdot 10 + 1}{20} = \frac{21}{20} = 1{,}05$$

$$K_s \cdot \frac{8F \cdot C}{\pi d^2} = 0{,}45 \cdot S_{ut}$$

$$K_s \cdot \frac{8F \cdot C}{\pi d^2} = 0{,}45 \cdot \frac{A}{d^m} = 0{,}45 \cdot \frac{2060}{d^{0{,}163}}$$

$$K_s \cdot \frac{8F \cdot C}{\pi} \cdot \frac{1}{0{,}45\,A} = d^{2-m}$$

$$1{,}05 \cdot \frac{8 \cdot 60 \cdot 10}{\pi} \cdot \frac{1}{0{,}45 \cdot 2060} = d^{2-0{,}163} \Rightarrow d = 1{,}348 \text{ mm}$$

$$D = 13{,}48 \text{ mm}$$

(8) First calculate fatigue stress conc. factors:

$K_{t_A} \approx 1.75 \qquad q \approx 0.9 \Rightarrow K_f = 1 + q(K_t - 1) = 1 + 0.9(1.75 - 1)$
$= 1.675$

$\frac{r}{d} = \frac{4}{25} = 0.16$

$\frac{D}{d} = \frac{35}{25} = 1.4$

$K_{t_B} \cong 2.4 \qquad q \approx 0.87 \qquad K_f = 1 + 0.87(2.4 - 1) = 2.218$

$\frac{r}{d} = \frac{3}{35-6} = \frac{3}{29} = 0.103$

$\frac{w}{d} = \frac{35}{29} = 1.207$

$K_{t_c} \cong 2.6 \qquad q \approx 0.85 \qquad K_f = 1 + 0.85(2.6 - 1) = 2.36$

$\frac{d}{w} = \frac{5}{35} = 0.143$

$S_{ut} = 770 \text{ MPa}$

$K_a = a \cdot S_{ut}^{b} = 57.7 \cdot 770^{-0.718} = 0.488$

$K_b = 1$

$K_c = 0.923$

$S_e = K_a \cdot K_b \cdot K_c \cdot (0.504 \cdot S_{ut}) = 0.488 \cdot 1 \cdot 0.923 \cdot 0.504 \cdot 770 = 175 \text{ MPa}$

Stresses :

Point A

$$\sigma_{min} = \frac{2000}{5\cdot 25} = 16\ MPa \qquad \sigma_{max} = \frac{10\,000}{5\cdot 25} = 80\,MPa$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{80-16}{2} = \frac{64}{2} = 32\ MPa$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{80+16}{2} = \frac{96}{2} = 48\ MPa$$

Point B

$$\sigma_{min} = \frac{2000}{5\cdot 29} = 13.79\ MPa \qquad \sigma_{max} = \frac{10000}{5\cdot 29} = 68.97\ MPa$$

$$\sigma_a = \frac{68.97 - 13.79}{2} = 27.59\ MPa \qquad \sigma_m = \frac{68.97 + 13.79}{2} = 41.38\,MPa$$

Point C

$$\sigma_{min} = \frac{2000}{30\cdot 5} = 13.33\ MPa \qquad \sigma_m = \frac{10000}{30.5} = 66.67\ MPa$$

$$\sigma_a = \frac{66.67 - 13.33}{2} = 26.67 \qquad \sigma_m = 40\ MPa$$

Modified Goodman

$$\frac{K_f \cdot \sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \Rightarrow n = \frac{1}{\frac{K_f \cdot \sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

Point A : $$n = \frac{1}{\frac{1.675 \cdot 32}{175} + \frac{48}{770}} = 2.7713$$

Point B : $$n = \frac{1}{\frac{2.218 \cdot 27.59}{175} + \frac{41.38}{770}} = 2.47$$

Point C : $n = \dfrac{1}{\dfrac{2.36 \cdot 26.67}{175} + \dfrac{40}{770}} = 2.429$

All points are safe; the component will not fail.