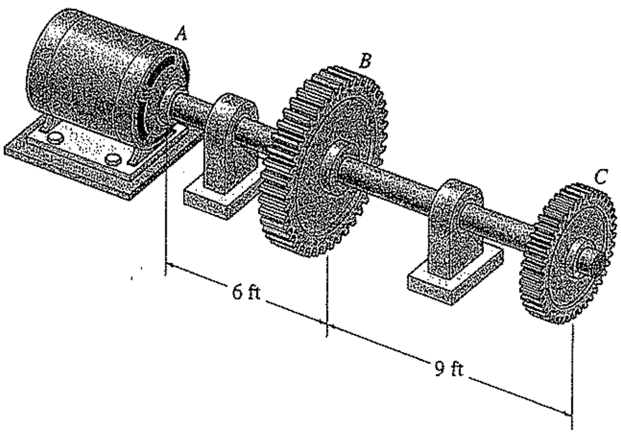
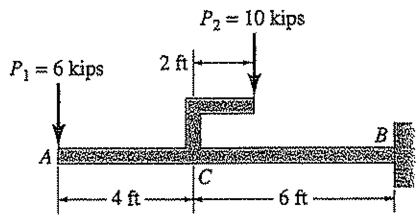
ME 325 – Homework #1
Due: Wed., Jan 28, 2009

1. The motor shown in the figure below delivers 95 hp at 600 rpm. Gears *B* and *C* transmit 55 hp and 40 hp, respectively, to operating machine tools. The shaft is made of steel having an allowable shear stress $\tau_{all} = 10$ ksi and a shear modulus of elasticity $G = 11.5 \times 10^6$ psi. Determine:

(a) the required diameter of segments *AB* and *BC* of the shaft.
(b) The corresponding angle of twist in degrees between the ends *A* and *C* of the shaft.



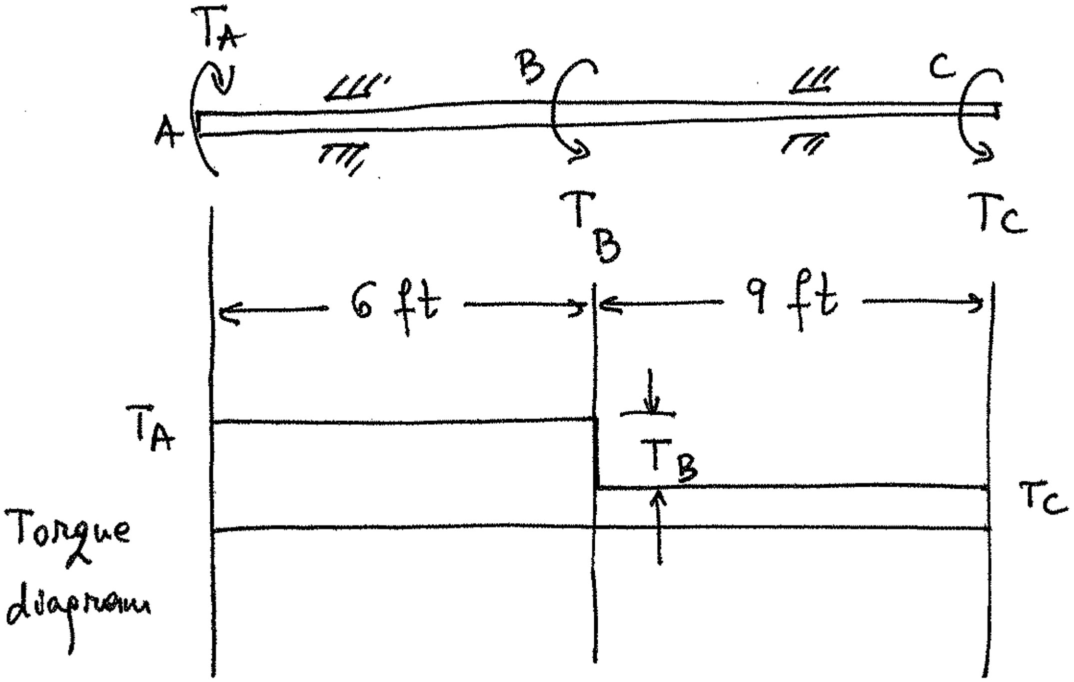
2. Determine the maximum normal stress and maximum shear stress in the beam *AB*. Assume a circular cross-section with a diameter of 3 in.



(1) $T_A = \dfrac{63\,000\,P}{n} = \dfrac{63000 \cdot 95}{600} = 9975 \text{ lb}\cdot\text{in}$

$T_B = \dfrac{55}{95}\,T = 5775 \text{ lb}\cdot\text{in}$

$T_C = \dfrac{40}{95}\,T = 4200 \text{ lb}\cdot\text{in}$



As it can be observed from the torque diagram above, the torques loading each segment are:

$T_{AB} = T_A = 9975 \text{ lb}\cdot\text{in}$

$T_{BC} = T_C = 4200 \text{ lb}\cdot\text{in}$

(a) Required diameters for each segment

$$\tau = \frac{T \cdot y}{J}$$

if $y = \frac{d}{2} \Rightarrow \tau = \tau_{max} \Rightarrow \tau_{max} = \dfrac{T \cdot \frac{d}{2}}{\frac{\pi d^4}{32}} = \dfrac{16\,T}{\pi d^3}$

OR,

$$d = \sqrt[3]{\frac{16T}{\pi \tau_{max}}}$$

$$d_{AB} = \sqrt[3]{\frac{16 \cdot 9975}{\pi \cdot 10^4}} = 1.719 \text{ in} \approx 1.72 \text{ in}$$

$$d_{BC} = \sqrt[3]{\frac{16 \cdot 4200}{\pi \cdot 10^4}} = 1.288 \text{ in} \approx 1.3 \text{ in}$$

(b) Angle of twist

$$\phi = \frac{T \cdot L}{G I_p}$$

$$\phi_{AC} = \phi_{AB} + \phi_{BC}$$

$$\phi_{AB} = \frac{T_{AB} \cdot L_{AB}}{G \cdot J_{AB}} = \frac{9975 \cdot 6 \cdot 12}{11.5 \cdot 10^6 \cdot \left[\frac{\pi \cdot (1.72)^4}{32}\right]} = 0.073 \text{ rad.}$$

$$\phi_{BC} = \frac{T_{BC} \cdot L_{BC}}{G \cdot J_{BC}} = \frac{4200 \cdot 9 \cdot 12}{11.5 \cdot 10^6 \left[\frac{\pi \cdot (1.3)^4}{32}\right]} = 0.141 \text{ rad}$$

$$\phi = 0.073 + 0.141 = 0.214 \text{ rad} = 12.26^\circ$$

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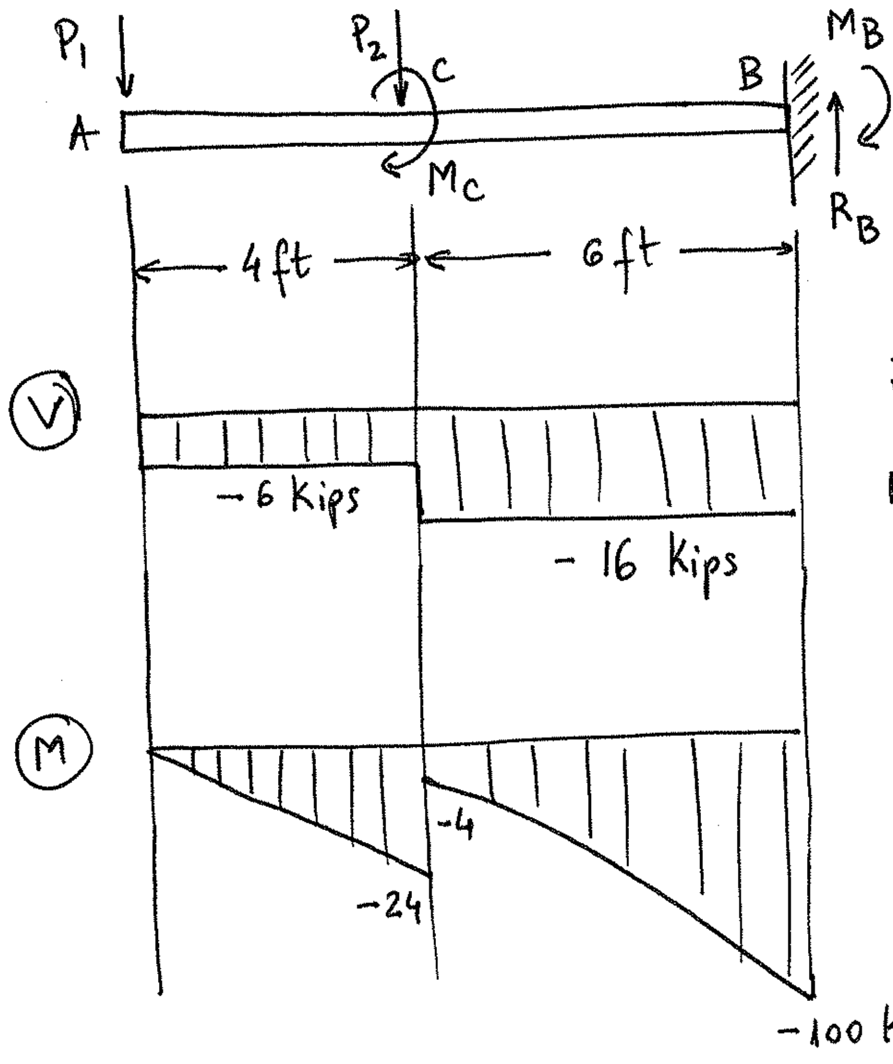


$M_C = P_2 \cdot 2\,ft =$

$= 10\ Kips \cdot 2\,ft$

$= 20\ Kips \cdot ft$

$R_B = P_1 + P_2 = 6 + 10 = 16\ kips$

$M_B = P_1 \cdot 10 + P_2 \cdot 6 - M_C =$

$= 6 \cdot 10 + 10 \cdot 6 - 20 =$

$= 100\ Kips \cdot ft$

Maximum normal stress is given by the maximum bending moment at location B.

$$\sigma = \frac{My}{I} = \frac{M \cdot \frac{d}{2}}{\frac{\pi d^4}{64}} = \frac{32M}{\pi d^3}$$

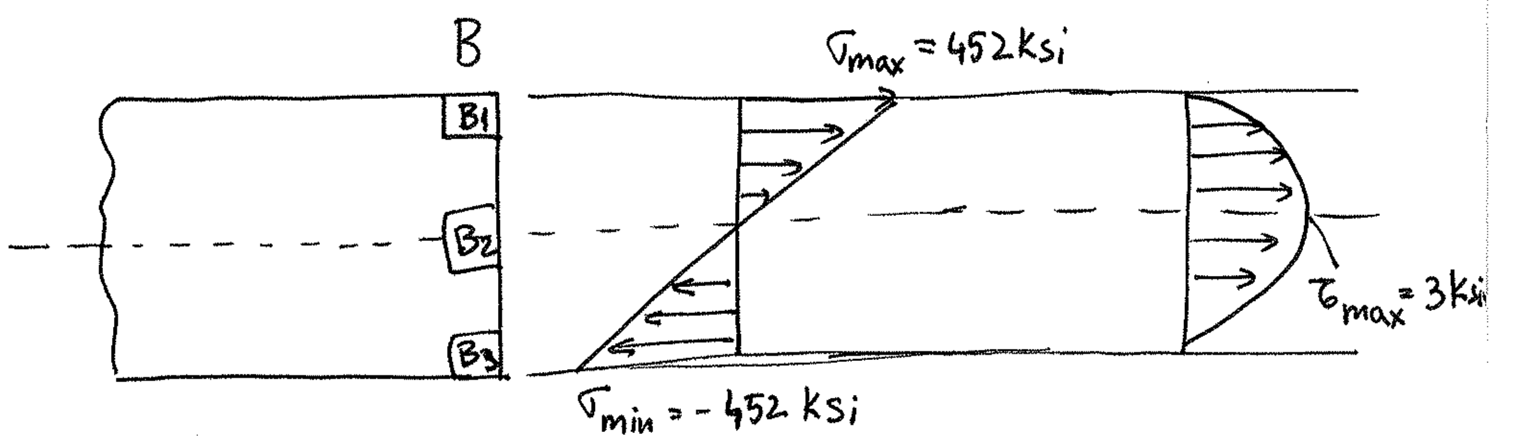
$$\sigma = \frac{32 \cdot 100 \cdot 10^3 \cdot 12\ lb \cdot in}{\pi \cdot 3^3\ in^3} = 452\ Ksi$$

(tension at the top, compression at the bottom)

Maximum shear stress occurs at any point between C and B, because the shear force is maximum in this section.

$$\tau_{max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{16\cdot 10^3}{\frac{\pi\, 3^2}{4}} = 3.02 \text{ Ksi}$$

The stress distribution at point B along the cross-section of the beam :



Stress states at points B_1, B_2 and B_3 :