ME 325 – Homework #3
Due: Fri., Feb. 13, 2009

1. Problem 7-23.
2. Problem 7-24.
3. Problem 7-27.
3. Problem 7-28.

(7-23)

$$K_a = a \cdot S_{ut}^{b} = 272 \cdot (1400)^{-0.995} = 0.201$$

$$K_b = \left(\frac{d_e}{7.62}\right)^{-0.1133} = \left(\frac{29.8}{7.62}\right)^{-0.1133} = 0.857$$

$$d_e = 0.808\sqrt{hb} = 0.808 \cdot \sqrt{75 \cdot 18} = 29.8 \text{ mm}$$

$$K_c = 1$$

$$K_d = 1$$

$$S_e = K_a K_b K_c K_d S_e' = 0.201 \cdot 0.857 \cdot (0.504)(1400) = 122 \text{ MPa}$$

Fig A-15-2 $K_t = 2.2$

$q = 0.95 \Rightarrow K_f = 1 + q(K_t - 1) = 1 + 0.95(2.2 - 1) = 2.12$

The maximum nominal stress is:

$$\sigma_{max} = \frac{My}{I} = \frac{800.25 \cdot \frac{18}{2}}{\frac{18^3}{12}(75-10)} = 0.228 \frac{\text{KN}}{\text{mm}^2} = 228 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 228 \text{ MPa}$$

$$M = \frac{F}{2} \cdot \frac{300}{2} = 75F$$

$$M_{max} = 75 \cdot F_{max} = 75 \cdot 10.670 = 800.25 \text{ KN} \cdot \text{mm}$$

$$M_{min} = 75 \cdot F_{min} = 75 \cdot 9.36 = 702 \text{ KN} \cdot \text{mm}$$

$$\sigma_{min} = \frac{702}{800.25}\sigma_{max} = 200 \text{ MPa}$$

Alternating and mean stresses :

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{228 - 200}{2} = 14 \text{ MPa}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{228 + 200}{2} = 214 \text{ MPa}$$

Applying modified Goodman equation

$$n = \frac{1}{\frac{K_f \sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = 2.52$$

7-24

$S_{ut} = 0.45\, HB = 0.45 \cdot 380 = 171$ ksi

$S_e' = 0.504 \cdot (171) = 86.2$ ksi

$K_a = a \cdot S_{ut}^{b} = 14.4 \cdot (171)^{-0.718} = 0.359$

$d_e = 0.37 \cdot D = 0.37 \cdot \left(\frac{3}{8}\right) = 0.139$ in (because the specimen is non-rotating)

$$K_b = \left(\frac{d_e}{0.3}\right)^{-0.1133} = \left(\frac{0.139}{0.3}\right)^{-0.1133} = 1.09$$

$S_e = 0.359 \cdot 1.09 \cdot 86.2 = 33.7$ ksi

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{30-15}{2} = 7.5 \text{ lb}$$

$$F_m = \frac{F_{max} + F_{min}}{2} = \frac{30+15}{2} = 22.5 \text{ lb}$$

$$\sigma_a = \frac{M_a \cdot y}{I} = \frac{M_a \cdot \frac{d}{2}}{\frac{\pi d^4}{64}} = \frac{32 M_a}{\pi d^3} = \frac{32 \cdot (7.5) \cdot 16}{\pi \left(\frac{3}{8}\right)^3} = 23.2 \text{ ksi}$$

$$\sigma_m = \frac{M_m}{M_a} \cdot \sigma_a = \frac{22.5}{7.5} \cdot \sigma_a = 69.5 \text{ ksi}$$

Find the fatigue strength of the component for finite life when $\sigma_m \neq 0$

$$\frac{\sigma_a}{S_f} + \frac{\sigma_m}{S_{ut}} = 1 \Rightarrow S_f = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}} = \frac{23.2}{1 - \frac{69.5}{171}} = 39.3 \text{ ksi}$$

And,

$$S_f = a \cdot N^b$$

$$a = \frac{(0.9 \cdot S_{ut})^2}{S_e} = \frac{(0.9 \cdot 171)^2}{33.7} = 703 \text{ ksi}$$

$$b = -\frac{1}{3} \log \frac{0.9(S_{ut})}{S_e} = -\frac{1}{3} \log \frac{0.9 \cdot 171}{33.7} = -0.22$$

$$N = \left(\frac{S_f}{a}\right)^{\frac{1}{b}} = \left(\frac{39.3}{703}\right)^{-\frac{1}{0.22}} = 494{,}000 \text{ cycles.}$$

(7-27)

(a) $\tau_m = 103\ MPa \quad \sigma_a = 172\ MPa$

$$\sigma_{ae} = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sigma_a = 172\ MPa$$

$$\sigma_{me} = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{3}\ \tau_m = 178\ MPa$$

$$\frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}} = \frac{1}{n} \Rightarrow n = \frac{1}{\frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}}} = \frac{1}{\frac{172}{276} + \frac{178}{551}} = 1.06$$

(b) $\tau_m = 138\ MPa \quad \tau_a = 69\ MPa$

$$\sigma_{ae} = \sqrt{3}\ \tau_a = 69\sqrt{3} = 119.5\ MPa$$

$$\sigma_{me} = \sqrt{3}\ \tau_m = \sqrt{3} \cdot 138 = 239\ MPa$$

$$n = \frac{1}{\frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}}} = \frac{1}{\frac{119.5}{276} + \frac{239}{551}} = 1.15 \text{ (fatigue)}$$

$$\tau_{max} = \tau_m + \tau_a = 138 + 69 = 207\ MPa$$

$$S_{ys} = 0.577 \cdot S_{yt} = \frac{S_{yt}}{\sqrt{3}} = 0.577 \cdot 413 = 238.01\ MPa$$

$$n = \frac{S_{ys}}{\tau_{max}} = \frac{238}{207} = 1.15 \text{ (static)}$$

(c) $\tau_m = 103$ MPa $\quad \tau_a = 69$ MPa $\quad \sigma_a = 83$ MPa

$$\sigma_{me} = \sqrt{3}\,\tau_m = 103\sqrt{3} = 178 \text{ MPa}$$

$$\sigma_{ae} = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{83^2 + 3 \cdot 69^2} = 146 \text{ MPa}$$

$$n = \frac{1}{\frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}}} = \frac{1}{\frac{146}{276} + \frac{178}{551}} = 1.17 \text{ (fatigue)}$$

$$\tau_{max} = \tau_m + \tau_a = 103 + 69 = 172 \text{ MPa}$$

$$\sigma_{max} = \sigma_a = 83 \text{ MPa}$$

$$\sigma_{max_e} = \sqrt{\sigma_{max}^2 + 3\tau_{max}^2} = \sqrt{83^2 + 3 \cdot 172^2} = 309 \text{ MPa}$$

$$n = \frac{S_{yt}}{\sigma_{max_e}} = \frac{413}{309} = 1.34 \text{ (static)}$$

(d) $\tau_a = 207$ MPa

$$S_{se} = 0.577 \cdot S_e = 0.577 \cdot (276) = 159 \text{ MPa}$$

$$n = \frac{S_{se}}{\tau_a} = \frac{159}{207} < 1 \rightarrow \text{failure}$$

Let's calulate the life of this component:

$$\sigma_{ae} = \sqrt{3}\,\tau_a = \sqrt{3} \cdot 207 = 359 \text{ MPa}$$

$$a = \frac{(0.9\,S_{ut})^2}{S_e} = \frac{(0.9 \cdot 551)^2}{276} = 891 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9\,S_{ut}}{S_e} = -\frac{1}{3}\log\frac{0.9 \cdot 551}{276} = -0.0848$$

$$N = \left(\frac{\sigma_{ae}}{a}\right)^{\frac{1}{b}} = \left(\frac{359}{891}\right)^{-\frac{1}{0.0848}} = 45{,}200 \text{ cycles}$$

(e) $\tau_a = 103$ MPa $\quad \sigma_m = 103$ MPa

$$\sigma_{ae} = \tau_a\sqrt{3} = 103\sqrt{3} = 178 \text{ MPa}$$

$$\sigma_{me} = \sigma_m = 103 \text{ MPa}$$

$$n = \frac{1}{\frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}}} = \frac{1}{\frac{178}{276} + \frac{103}{551}} = 1.20 \text{ (fatigue)}$$

$$\sigma_{max} = \sigma_m = 103 \text{ MPa}$$

$$\tau_{max} = \tau_a = 103 \text{ MPa}$$

$$\sigma_{max_e} = \sqrt{\sigma_{max}^2 + 3\tau_{max}^2} = \sqrt{103^2 + 3 \cdot 103^2} = 206 \text{ MPa}$$

$$n = \frac{S_{yt}}{\sigma_{max}} = \frac{\cancel{206}}{} = \frac{413}{206} = 2.0 \text{ (static)}$$

7-28 $d = 600\ mm$

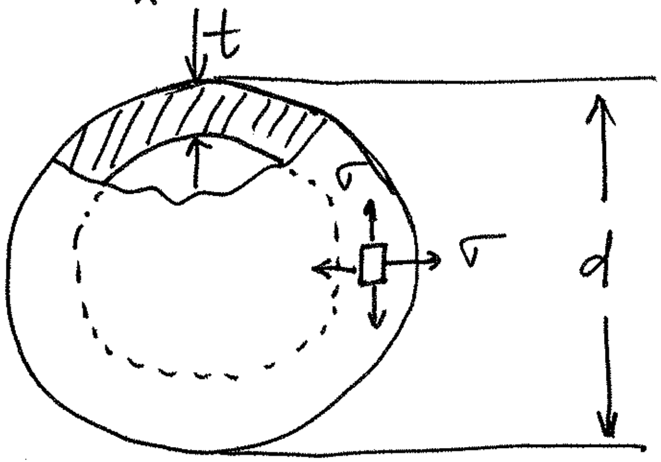
cold-drawn steel $S_{ut} = 440\ MPa$ $S_y = 370\ MPa$

$t = 3mm$

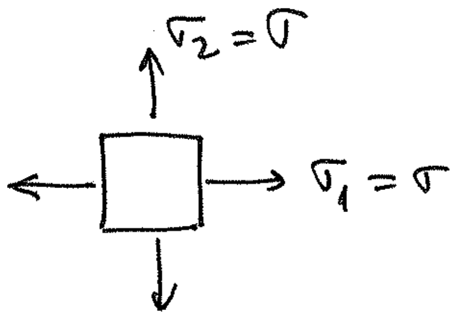
(a) For static yielding

$$\sigma_{eq} = S_y$$

where σ_{eq} = von Mises stress



The stress state in the skin of the pressure vessel is biaxial



$$\sigma = \frac{pd}{4t}$$

$$\sigma_{eq} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} = \sqrt{\sigma^2 + \sigma^2 - \sigma^2} = \sigma = \frac{pd}{4t}$$

$$\frac{pd}{4t} = S_y \Rightarrow p = \frac{4 \cdot S_y \cdot t}{d} = \frac{4 \cdot 370 \cdot 3}{600} = 7.4\ MPa$$

(b) $\sigma_{min} = 0$

$\sigma_{max} = \frac{P_{max} \cdot d}{4t}$ $\Bigg\} \Rightarrow$ $\sigma_a = \frac{1}{2}\sigma_{max} = \frac{P_{max} \cdot d}{8t}$

$\sigma_m = \sigma_a = \frac{1}{2}\sigma_{max} = \frac{P_{max} \cdot d}{8 \cdot t}$

According to von Mises formula:

$$\sigma_{eq_a} = \sigma_a = \frac{P_{max} \cdot d}{8t}$$

$$\sigma_{eq_m} = \sigma_m = \frac{P_{max} \cdot d}{8t}$$

Goodman relation:

$$\frac{\sigma_{eq_a}}{S_e} + \frac{\sigma_{eq_m}}{S_{ut}} = 1$$

Compute S_e

$$S_e = K_a \cdot K_b \cdot K_c \cdot K_d \cdot k_e \cdot S_e'$$

$$S_e' = 0.504 \cdot S_{ut} = 0.504 \cdot 440 = 221.76 \text{ MPa}$$

$$K_a = a \cdot S_{ut}^{b} = 4.51 \cdot 440^{-0.265} = 0.899$$

$K_b \cong 0.6$ (we used the fact that the cross-section is round with $d = 600$ mm, and the stresses are distributed uniformly around the circumference → similar to a rotating specimen)

$K_c = 0.923$ (tension – the cross-section of the wall is loaded with a constant stress)

$K_d = 1$

$K_e = 1$

$$S_e = 0.6 \cdot 0.899 \cdot 221.76 = 120 \text{ MPa}$$

Going back to the Goodman equation

$$\frac{\frac{P_{max} \cdot d}{8t}}{S_e} + \frac{\frac{P_{max} \cdot d}{8t}}{S_{ut}} = 1 \iff$$

$$P_{max}\left[\frac{1}{S_e} + \frac{1}{S_{ut}}\right]\frac{d}{8t} = 1 \implies$$

$$\implies P_{max} = \frac{8t}{d\left[\frac{1}{S_e} + \frac{1}{S_{ut}}\right]} = \frac{8 \cdot 3}{600\left[\frac{1}{120} + \frac{1}{440}\right]} = 3.8 \text{ MPa}$$