ME 325 – Homework #4
Due: Mon., Mar. 9, 2009

1. Problem 8-11.
2. Problem 8-16.
3. Problem 8-20.
4. Problem 8-27.
5. Problem 8-28.

① Problem 8-11

External load:

$$P = \frac{\pi}{4} d^2 \cdot p$$

where d = effective sealing diameter
and p = internal gas pressure

$$P = \frac{\pi}{4} \cdot (150)^2 \cdot 6 = 106 \text{ KN}$$

Static force per bolt:

$$\frac{P}{10} = 10.6 \text{ KN}$$

$L_G = D + E = 45$ mm $\quad d = 12$ mm

$H = 10.8$ mm , no washers specified

$L_T = 2D + 6 = 2 \cdot 12 + 6 = 30$ mm

$L_G + H = 45 + 10.8 = 55.8$ mm

Table A-17: $L = 60$ mm

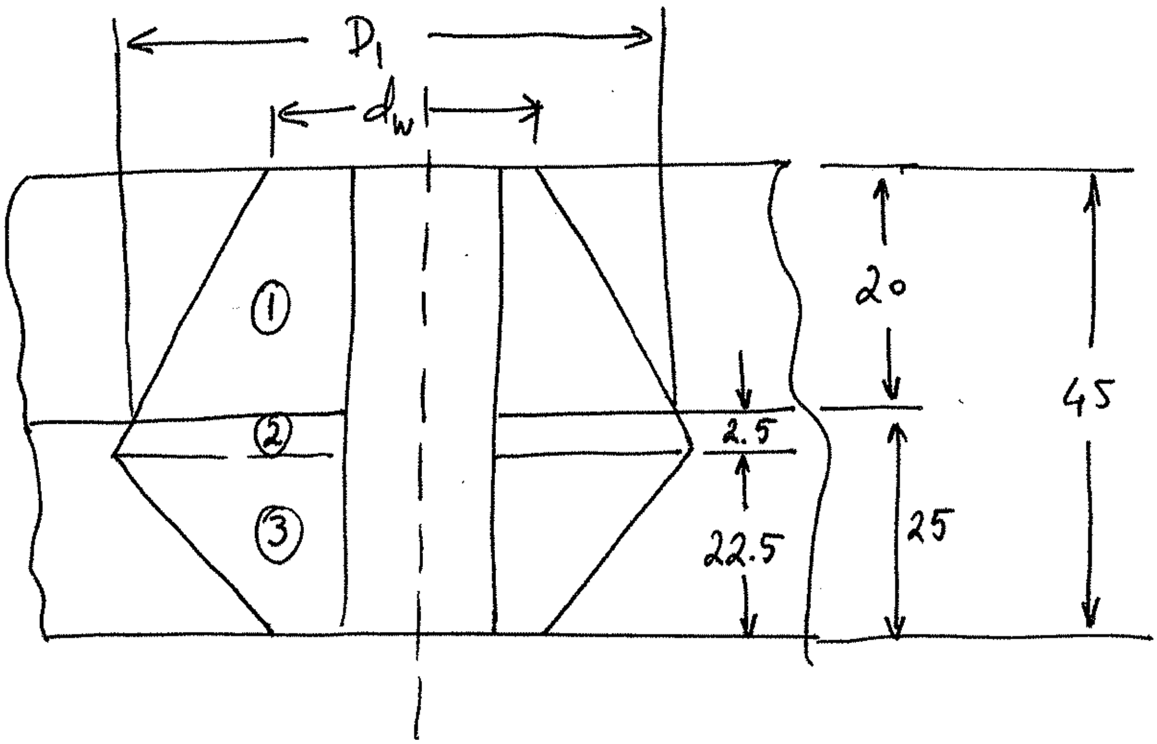
$l_d = L - L_T = 60 - 30 = 30$ mm

$l_T = 45 - 30 = 15$ mm

$$A_d = \frac{\pi (12)^2}{4} = 113 \text{ mm}^2$$

Table 8-1 : $A_t = 84.3\ mm^2$

$$K_b = \frac{A_d A_t \cdot E}{A_d \cdot l_T + A_t \cdot l_d} = \frac{113 \cdot (84.3)(207)}{113 \cdot (15) + 84.3(30)} = 466.8 \frac{MN}{m}$$



Three frustra $d_w = 1.5 \cdot (12) = 18\ mm$

$D_1 = d_w + 2 \cdot (20) \cdot \tan 30^\circ = 41.09\ mm$

$$K = \frac{0.577 \pi E d}{\ln \frac{(1.15t + D - d)(D + d)}{(1.15t + D + d)(D - d)}}$$

1-st frustrum $t = 20\ mm$, $D = d_w = 18\ mm$, $d = 12\ mm$, $E = 207 GPa$

$K_1 = 4470 \frac{MN}{m}$

2-nd frustum: $t = 2.5\ mm$, $D = D_1 = 41.09\ mm$, $d = 12\ mm$

$E = 113\ GPa$

$$K_2 = 59\,040\ \frac{MN}{m}$$

3-rd frustum: $t = 22.5\ mm$, $D = d_W = 18\ mm$, $d = 12\ mm$

$E = 113\ GPa$

$$K_3 = 2343\ \frac{MN}{m}$$

$$K_m = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}} = 1498\ \frac{MN}{m}$$

$$C = \frac{K_b}{K_b + K_m} = \frac{466.8}{466.8 + 1498} = 0.238$$

Tables 8-1 and 8-6: $A_t = 84.3\ mm^2$; $S_p = 600\ MPa$

$$F_i = 0.75 \cdot S_p \cdot A_t = 0.75 \cdot 600 \cdot 84.3 = 37.9\ KN$$

$$n = \frac{S_p \cdot A_t - F_i}{C \cdot \frac{P}{10}} = \frac{600 \cdot 84.3 - 37.9 \cdot 10^3}{0.238 \cdot 10.6 \cdot 10^3} = 5.03$$

Check the factor of safety against separation:

$$n_{separation} = \frac{F_i}{(1-C)P} = \frac{37.9}{(1-0.238) \cdot 10.6} = 4.7$$

OK separation does not occur.

(2) Figure 8-18 :

$t_1 = 0.25$ in

$h = 0.25 + 0.065 = 0.315$ in

$l = h + \frac{d}{2} = 0.315 + \frac{3}{16} = 0.5025$ in

$D_1 = d_w + l \cdot \tan\alpha = 1.5 \cdot \left(\frac{3}{8}\right) + 0.5025 \cdot \tan 30° = 0.8524$ in

$D_2 = d_w = 1.5 \cdot d = 1.5 \cdot (0.375) = 0.5625$ in

3 fustra :

(1) Washer : $E = 30$ Mpsi ; $t = 0.065$; $D = 0.5625$

$K_1 = 78.58 \ \frac{Mlb}{in}$

(2) Cap : $E = 14$ Mpsi ; $t = \frac{l}{2} - t_{washer} = \frac{0.5025}{2} - 0.065 = 0.18625$

$D = d_w + \frac{l}{2} \cdot \tan 30° = 0.5625 + \frac{0.5025}{2} \cdot \tan 30° = 0.8524$

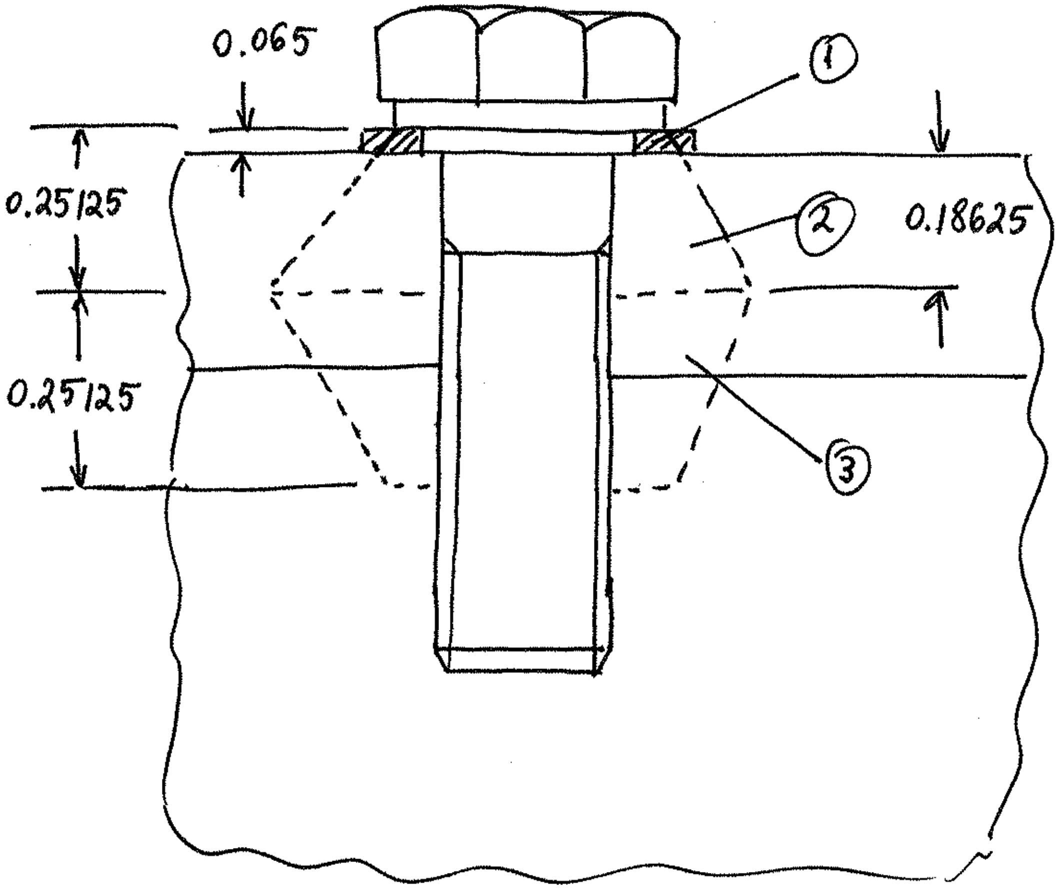
$K_2 = 23.428 \ \frac{Mlb}{in}$

(3) Cap and frame : $E = 14$ Mpsi ; $t = \frac{l}{2} = 0.25125$ mm

$D = d_w = 0.5625$ mm

$k_3 = 14.31 \ \frac{Mlb}{in}$

See next page the 3 fustra :



$$K_m = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}} = 7.98 \ \frac{Mlb}{in}$$

For bolt :

$$L_T = 2 \cdot \frac{3}{8} + \frac{1}{4} = 1 \ in \qquad A_t = 0.0775 \ in^2$$

It is threaded all the way

$$K_b = \frac{A_t \cdot E}{l} = \frac{0.0775 \cdot 30}{0.5025} = 4.63 \ \frac{Mlb}{in}$$

③ Problem 8-20

(a) $A_t = 245\ mm^2 \quad S_p = 600\ MPa$

$$F_i = 0.9 \cdot S_p \cdot A_t = 0.9 \cdot 245 \cdot 600 = 132.3\ kN$$

$$T = 0.18 \cdot F_i \cdot d = 0.18 \cdot 132.3 \cdot (20) = 476\ N \cdot m$$

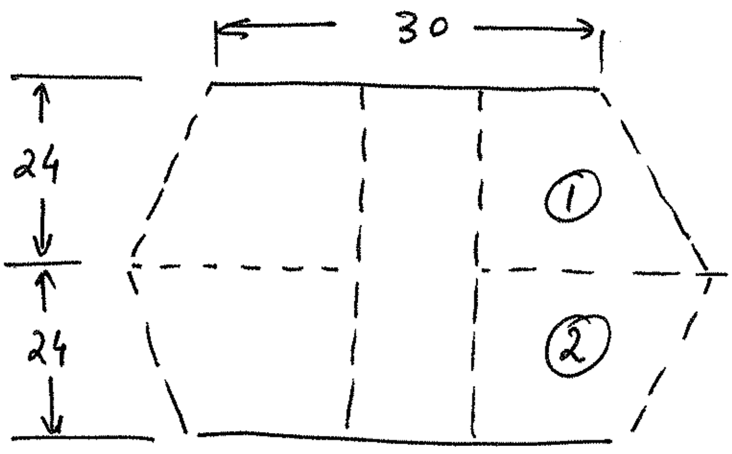
(b) $L > 48 + 18 = 66\ mm \Rightarrow$ use $L = 80\ mm$

$L_T = 2D + 6 = 2 \cdot 20 + 6 = 46\ mm \Rightarrow l_d = 34\ mm \Rightarrow$

$l_T = 14\ mm$

$$k_b = \frac{A_d A_t E}{A_d l_T + A_t \cdot l_d} = \frac{314.1 \cdot 245 \cdot 207 \cdot 10^3}{314.1 \cdot 14 + 245 \cdot 34} = 1252\ \frac{MN}{m}$$

$$A_d = \frac{\pi \cdot (20)^2}{4} = 314.1\ mm^2$$



$d_w = 1.5d = 30\ mm$

$$k_1 = k_2 = 8471\ \frac{MN}{m}$$

$$k_m = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} = \frac{k_1}{2} = 4235\ \frac{MN}{m}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{1252}{1252 + 4235} = 0.228$$

$$F_b = C \cdot P + F_i = 0.228 \cdot 20 + 132.3 = 137 \text{ KN}$$

$$F_m = (1-C)P - F_i = (1-0.228) \cdot 20 - 132.3 = -116.9 \text{ KN}$$

(4) Problem 8-27

(a) $A_t = 0.0775 \text{ in}^2$ $S_p = 85$ Ksi $S_{ut} = 120$ Ksi $S_e = 18.6$ Ksi

We will assume that the entire bolt within the grip is unthreaded. Then:

$$K_b = \frac{A_d \cdot E}{l} = \frac{\frac{\pi}{4}\left(\frac{3}{8}\right)^2 \cdot 30}{13.5} = 0.245 \ \frac{Mlb}{in}$$

Cross-sectional area of the cylinder is:

$$A = \frac{\pi}{4}\left[(D+2t)^2 - D^2\right] = \frac{\pi}{4}\left[\left(4+2\cdot\frac{3}{8}\right)^2 - 4^2\right] = 5.154 \text{ in}^2$$

$$k_m = \frac{AE}{l} = \frac{5.154 \cdot 30}{12} \cdot \frac{1}{6} = 2.148 \ \frac{Mlb}{in} \text{ (per each bolt)}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{0.245}{0.245 + 2.148} = 0.102$$

(b) $F_i = 0.75 \cdot S_p \cdot A_t = 0.75 \cdot 85 \cdot 0.0775 = 4.94$ kip

$$\sigma_i = 0.75 \cdot S_p = 0.75 \cdot 85 = 63.75 \text{ kpsi}$$

$$P = p \cdot A = 2000 \cdot \frac{\pi \cdot 4^2}{4} \cdot \frac{1}{6} = 4189 \text{ lb (per each bolt)}$$

$$\sigma_a = \frac{C \cdot P}{2A_t} = \frac{0.102 \cdot 4189}{2 \cdot (0.0775)} = 2.76 \text{ kpsi}$$

Goodman theory: $\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}} = \dfrac{1}{n}$

The stress in the bolt is

$$\sigma_b = \frac{F_b}{A_t} = \frac{F_i + CP}{A_t} = \frac{F_i}{A_i} + C\frac{P}{A_t} = \sigma_i + C\frac{P}{A_t}$$

The pressure fluctuates between 0 and $p_{max} = 2000$ psi

$$\left.\begin{aligned} \sigma_{b_{max}} &= \sigma_i + C\frac{P_{max}}{A_t} \\ \sigma_{b_{min}} &= \sigma_i + C\cdot\frac{0}{A_t} = \sigma_i \end{aligned}\right\} \rightarrow$$

$$\rightarrow \sigma_a = \frac{\sigma_{b_{max}} - \sigma_{b_{min}}}{2} = C\frac{P_{max}}{2A_t} = 2.76 \text{ ksi}$$

$$\sigma_m = \frac{\sigma_{b_{max}} + \sigma_{b_{min}}}{2} = \sigma_i + C\frac{P_{max}}{2A_t} = 63.75 + 2.76$$

$$= 66.51 \text{ ksi}$$

$$n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{1}{\frac{2.76}{18.6} + \frac{66.51}{120}} = 1.43$$

(c) The factor of safety against separation is

$$n = \frac{F_i}{P(1-C)} \Rightarrow \text{at separation } n = 1$$

$$P = \frac{F_i}{1-C} = \frac{4.94}{1-0.102} = 5.50 \text{ kip}$$

$$p = \frac{P}{A} = \frac{5.5\cdot 10^3\cdot 6}{\frac{\pi}{4}\cdot 4^2} = 2630 \text{ psi}$$

⑤ Problem 8-28

Members: $S_y = 71$ Ksi

$S_{sy} = 0.577 \cdot S_y = 0.577(71) = 41$ Ksi

Bolts: $S_y = 130$ Ksi

$S_{sy} = 0.577 \cdot (130) = 75.01$ ksi

Shear of bolts

$$A_s = \frac{2 \cdot \pi (0.375)^2}{4} = 0.221 \text{ in}^2$$

$$F_s = \frac{A_s \cdot S_{sy}}{n} = \frac{0.221 \cdot 75.01}{3} = 5.53 \text{ kip}$$

Bearing on bolts

$$A_b = 2 \cdot \frac{3}{8} \cdot \frac{1}{4} = 0.188 \text{ in}^2$$

$$F_b = \frac{A_b \cdot S_y}{n} = \frac{0.188 \cdot 130}{2} = 12.22 \text{ Kip}$$

Bearing on member

$$F_b = \frac{A_b \cdot S_y}{n} = \frac{0.188 \cdot 71}{2.5} = 5.34 \text{ Kip}$$

Tension of members

$$A_t = \left(\frac{5}{4} - \frac{3}{8}\right)\frac{1}{4} = 0.219 \text{ in}^2$$

$$F_t = \frac{A_t \cdot S_y}{n} = \frac{0.219 \cdot 71}{3} = 5.18 \text{ kip} \Rightarrow$$

F = 5.18 kip based on tension of members