ME 461 Fatigue and Fracture
Final exam
Thursday May 14, 2009

1. *(5 pts.)* Explain the difference between stress intensity factor and stress concentration factor, and give an example when each of these parameters is used.

Stress intensity factor is applied to cracked components
Stress concentration factor is applied to components that have notches (holes, fillets, grooves, etc)

2. *(5 pts.)* What is newest design methodology used these days in fatigue design and briefly explain the main feature of this approach.

The newest fatigue design methodology is the damage tolerant approach. This approach assumes that components have pre-existent cracks, and fatigue life is given by the number of cycles to grow an initial crack to a critical length.

3. *(5 pts.)* Explain what are persistent slip bands and if they are detrimental or beneficial to the fatigue life of a metallic component.

Persistent slip bands (PSB's) are intrusions- extrusions (ledges) at the outer surface of a specimen. They are caused by the back and forth slip of crystallographic planes with respect to each other. They are detrimental and should be removed to enhance fatigue life of components.

4. *(5 pts.)* What is the basic assumption of LEFM?

The basic assumption of LEFM is that components are essentially elastic, and the plasticity at the crack tip is very limited. Plastic zone size is small compared with the crack length and specimen's dimensions.

5. *(20 pts.)* A 1045 Annealed steel rod is subjected to a fluctuating constant amplitude strain. Using a strain-based approach, determine the strain range that will cause the failure of the specimen after 50,000 cycles.

$$\varepsilon_a = \frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^{b'} + \varepsilon_f'(2N_f)^{c}$$

$$b' = b + 0.159 \log K_a$$

$\sigma_f' = 916$ MPa $\quad \varepsilon_f' = 0.486 \quad n' = 0.152 \quad S_u = 752$ MPa

$E = 207$ GPa

$$b = -\frac{n'}{1+5n'} = -0.086 \qquad c = -\frac{1}{1+5n'} = -0.568$$

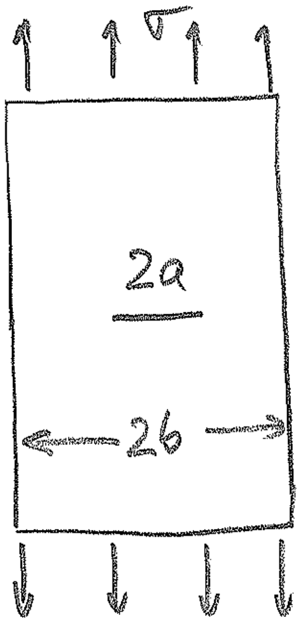
$$K_a = 4.51 \cdot 752^{-0.265} = 0.779$$

$$b' = -0.086 + 0.159 \log 0.779 = -0.103$$

$$\frac{\Delta\varepsilon}{2} = \frac{916}{270000}\left[(2\cdot 50{,}000)^{-0.103} + 0.482\,(2\cdot 50{,}000)^{-0.568}\right]$$

$$= 1.733\cdot 10^{-3} \Rightarrow \boxed{\Delta\varepsilon = 3.466\cdot 10^{-3}}$$

6. *(20 pts.)* A 7075-T6 middle tension specimen (MT) with the width $2b = 40$ cm and an initial crack length of $2a = 5$ cm is subjected to a stress that fluctuates between $S_{min} = 50$ MPa and $S_{max} = 250$ MPa. Determine the total amount of crack growth after 3 loading cycles.



7075-T6

$A = 2.7 \cdot 10^{-11} \frac{m}{cycle}$

$n = 3.7$

$\sigma_{max} = 250$ MPa $\quad \sigma_{min} = 50$ MPa

$R = \frac{\sigma_{min}}{\sigma_{max}} = 0.2 \quad \lambda \approx 0.5$

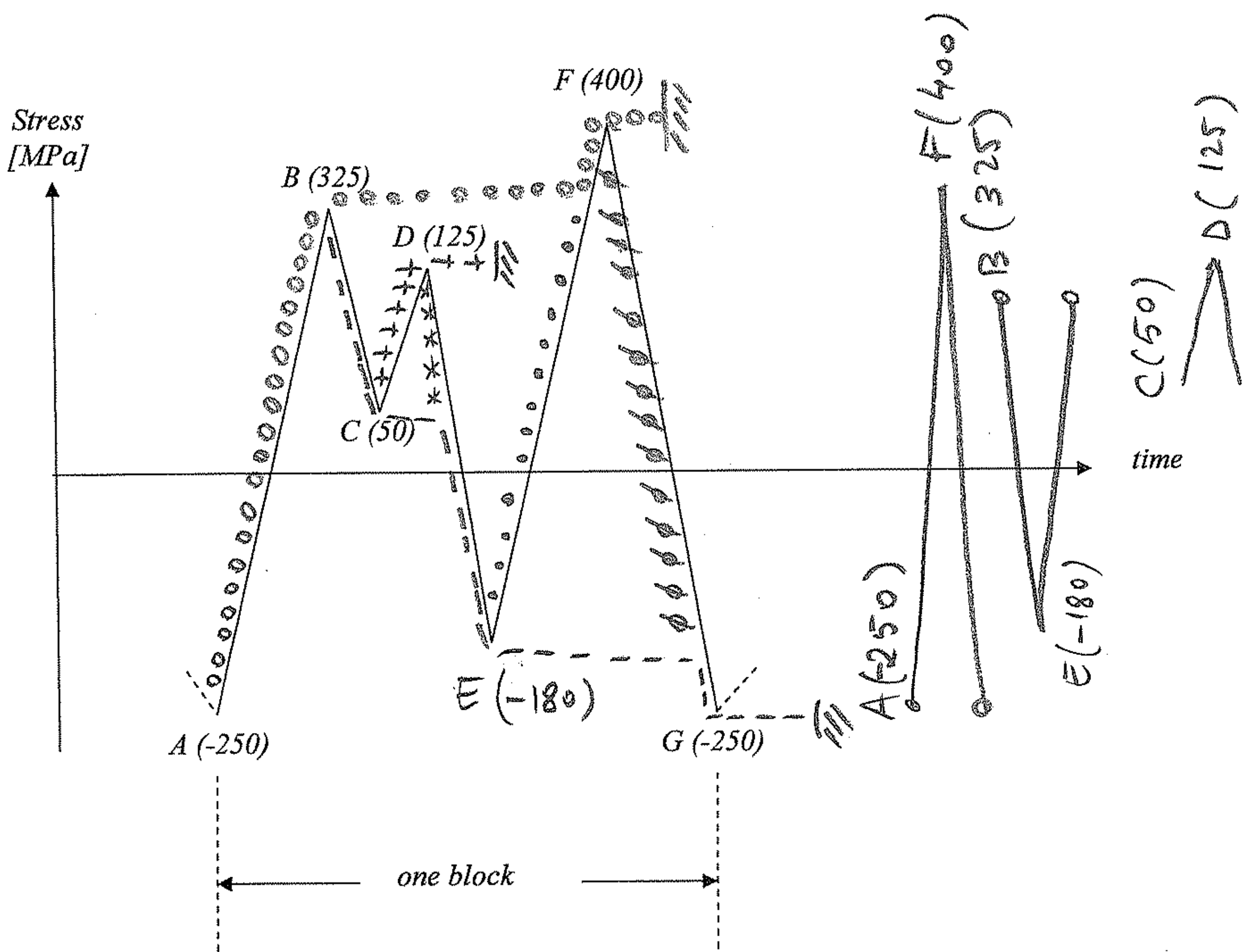
$\frac{da}{dN} = A''(\Delta K)^n \qquad A'' = \frac{A}{(1-R)^{n(1-\lambda)}} = \frac{2.7 \cdot 10^{-11}}{(1-0.2)^{3.7(0.5)}} = 4.08 \cdot 10^{-11} \frac{m}{cycle}$

$K = \sigma \sqrt{\pi a} \cdot F\left(\frac{a}{b}\right) \qquad F\left(\frac{a}{b}\right) = \sqrt{\frac{2b}{\pi a} \cdot \tan \frac{\pi a}{2b}}$

Cycle	a_0 [m]	$F\left(\frac{a}{b}\right)$	K_{max}	K_{min}	ΔK	da	$a_f = a + da$
1	0.02500	1.007	70.518	14.104	56.415	$1.23 \cdot 10^{-4}$	0.02513
2	0.02513	1.007	70.696	14.139	56.557	$1.24 \cdot 10^{-4}$	0.02524
3	0.025247	1.007	70.875	14.175	56.7	$1.256 \cdot 10^{-4}$	0.025373
Units	[m]	—	MPa$\sqrt{m}$	MPa$\sqrt{m}$	MPa$\sqrt{m}$	m	m

$$\boxed{\Delta a = a_f - a_0 = 373\ \mu m}$$

7. *(40 pts.)* A component made of aluminum alloy *2219-T851* is loaded with a variable amplitude loading according to the diagram below in which each loading block is repeated until failure. For this material $S_{ut} = 500\ MPa$, and $S_f = 120\ MPa$ at $N_f = 10^8$ cycles. The component is smooth ($k_t = 1$). Using the rainflow technique divide the loading block into independent loading cycles, and determine using the linear damage accumulation rule how many blocks will take until the failure of the component.



Aluminum 2219 - T851 $\quad S_u = 500$ MPa $\quad S_f = 120$ MPa at $N_f = 10^8$ cycles

$$\left.\begin{aligned} N_f = 1 &\Rightarrow S_{N_f} = S_{ut} \\ N_f = 10^8 &\Rightarrow S_{N_f} = S_f \end{aligned}\right\} \Rightarrow \quad S_{N_f} = a \cdot N_f^{\,b} \Rightarrow$$

$$a = S_{ut} = 500 \text{ MPa}$$

$$b = \frac{1}{8} \log \frac{S_f}{S_{ut}} = -0{,}077$$

Load segment	S_{min} (MPa)	S_{max} (MPa)	S_a (MPa)	S_m (MPa)	n	S_{N_f}	N_f
1.	−250	400	325	75	1	382	32.6
2.	−180	325	252.5	72.5	1	295	932.8
3.	50	125	37.5	87.5	1	45.4	$3.34 \cdot 10^3$

$$\frac{S_a}{N_f} + \frac{S_m}{S_{ut}} = 1 \Rightarrow \quad S_{N_f} = \frac{S_a}{1 - \frac{S_m}{S_{ut}}}$$

$$S_{N_f} = a \cdot N_f^{b} \Rightarrow \quad N_f = \left(\frac{S_{N_f}}{a}\right)^{\frac{1}{b}}$$

$$d_1 = \frac{n}{N_{f1}} = \frac{1}{32.6} = 0.031$$

$$d_2 = \frac{n}{N_{f2}} = \frac{1}{932.8} = 1.072 \cdot 10^{-3}$$

$$d_3 = \frac{n}{N_{f3}} = \frac{1}{334 \cdot 10^3} = 2.9 \cdot 10^{-4}$$

$$d = d_1 + d_2 + d_3 = 0.032$$

$N = \frac{1}{d} = 31.17$ blocks $\simeq$ 31 loading blocks before failure.