

1. (5 pts.) What is the basic assumption of LEFM?

The basic assumption of Linear Elastic Fracture Mechanics (LEFM) is that the plastic zone at the crack tip is very small, thus it can be neglected.

2. (5 pts.) What is stress intensity factor?

Stress intensity factor is a parameter that governs the stress field around a crack tip. It basically quantifies how fast the elastic stress increases while approaching the crack tip.

3. (5 pts.) List several common methods of estimating residual stress fields in components.

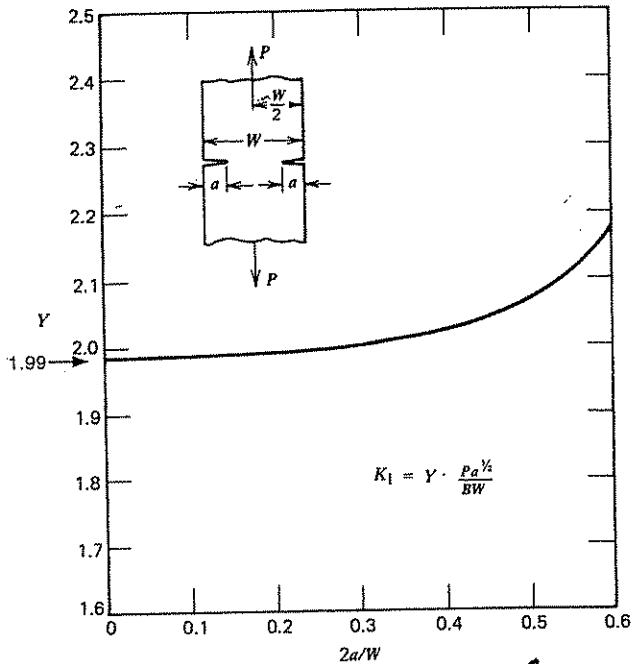
- analytical methods
- numerical (FEA)
- experimental: hole-drilling, sectioning, X-Ray diffraction, ultrasonic method.

4. (5 pts.) Increasing load ratio R while keeping ΔK constant leads to

- a) increased
- b) decreased
- c) unchanged

crack growth rate in a fatigue crack growth experiment in a metallic component.

5. (25 pts.) The specimen shown below is made of Ti-6Al-4V recrystallized. If the applied load is 10 kN, plate width is 5 cm and thickness is 2.5 mm, what is the critical crack length ensuring a factor of safety of 2 against failure by fracture?



Assume $Y = 1.99$ (small crack)

At fracture

$$K_I = \frac{K_{Ic}}{FS}$$

$$FS = 2$$

$$K_I = Y \frac{P \cdot a^{3/2}}{B W} = \frac{K_{Ic}}{FS} \Rightarrow$$

$$\Rightarrow a_{cr} = \frac{(K_{Ic} \cdot B \cdot W)^2}{(Y \cdot FS \cdot P)^2} = \frac{(85 \cdot 10^6 \cdot 2.5 \cdot 10^{-3} \cdot 5 \cdot 10^{-2})^2}{(1.99 \cdot 2 \cdot 10 \cdot 10^3)^2} = 0.07 \text{ m}$$

(crack larger than plate's width)

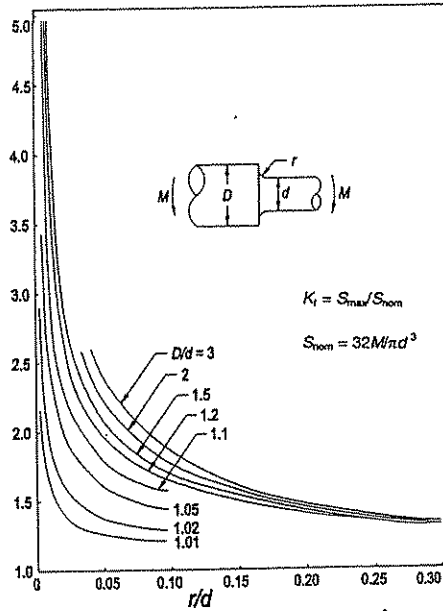
Now, try $Y \approx 2.18$ or $\frac{2a}{W} = 0.6$

$$a_{cr} = \frac{(85 \cdot 10^6 \cdot 2.5 \cdot 10^{-3} \cdot 5 \cdot 10^{-2})^2}{(2.18 \cdot 2 \cdot 10 \cdot 10^3)^2} = 0.059$$

So, the conclusion is that the critical crack length will be larger than $0.3 W$

$$a_{cr} > 0.3 W$$

6. (30 pts.) For the specimen shown in the figure below analytically calculate the number of cycles to failure (N_f) using the strain life equation, given that the following quantities are known: M_{max} the maximum applied bending moment in a cycle ($R = -1$), specimen's geometrical dimensions r , d and D . For strain hardening use cyclic strain strength coefficient k' , cyclic strain hardening exponent $n' = 1$. For the strain-life model use fatigue ductility exponent ϵ_f' and fatigue ductility exponent c . Assume the fatigue strength exponent b and fatigue strength coefficient σ_f' are negligible. Also, assume that cyclic yielding in the component occurs only locally.



Using Neuber's criterion:

$$K_\epsilon \cdot K_\sigma = k_f^2$$

In terms of amplitudes:

$$\frac{\sigma_a^2}{E} + \sigma_a \cdot \left(\frac{\sigma_a}{k'}\right)^{\frac{1}{n'}} = \frac{(k_f \cdot S)^2}{E}$$

Since $n' = 1$, factoring out σ_a

$$\sigma_a^2 \left[\frac{1}{E} + \frac{1}{k'} \right] = \frac{(k_f \cdot S)^2}{E} \Rightarrow \sigma_a^2 = \frac{k' (k_f \cdot S)^2}{E + k'} \Rightarrow \sigma_a = k_f S_a \sqrt{\frac{k'}{k' + E}}$$

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{k'}\right)^{\frac{1}{n'}} = \frac{\sigma_a}{E} + \frac{\sigma_a}{k'} = \sigma_a \frac{E + k'}{E k'} \quad \text{OR,}$$

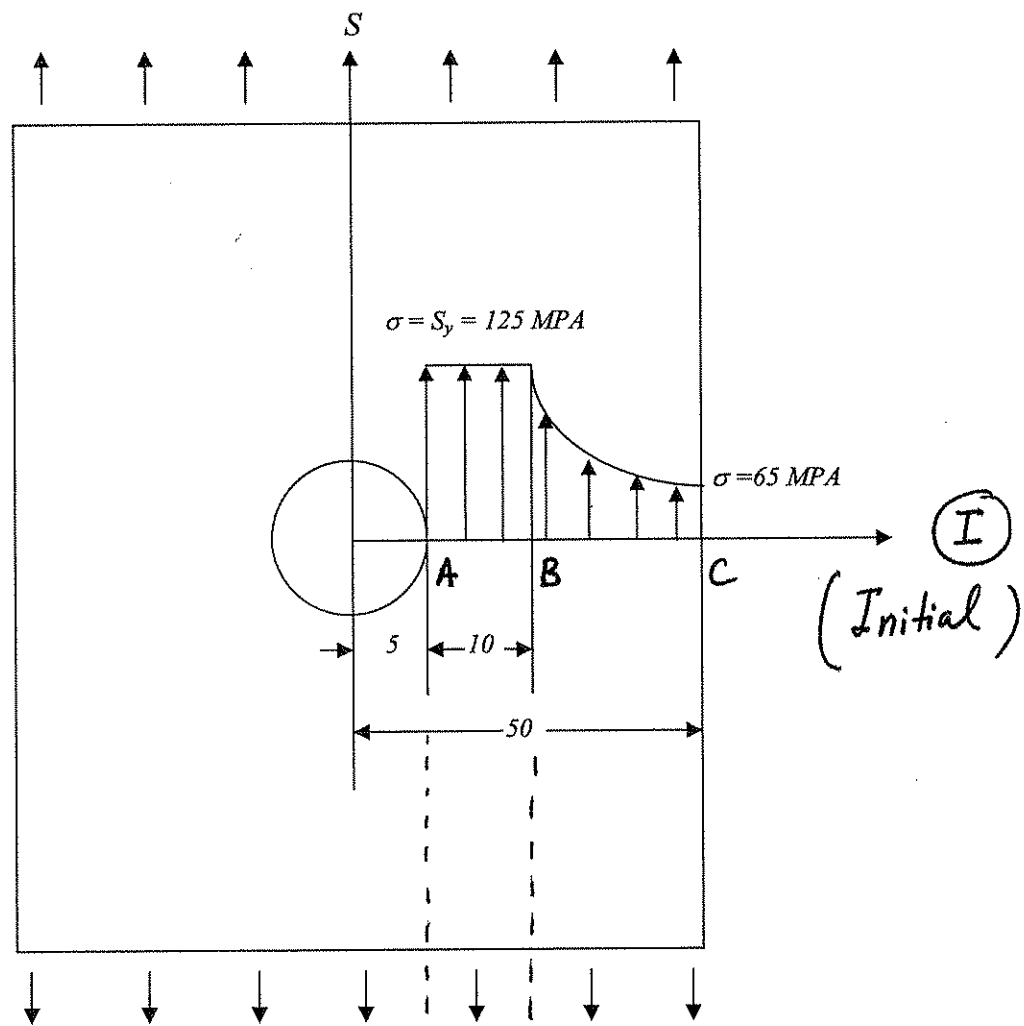
$$\epsilon_a = \frac{k_f S_a}{E} \sqrt{\frac{E + k'}{k'}}$$

Strain life equation with σ_f' and b negligible:

$$\epsilon_a = \epsilon_f' (2N)^c \Rightarrow N = \frac{1}{2} \left(\frac{\epsilon}{\epsilon_f'} \right)^{\frac{1}{c}} \quad \text{OR,}$$

$$N = \frac{1}{2} \left[\frac{k_f S_a}{E \cdot \epsilon_f'} \sqrt{\frac{E + k'}{k'}} \right]^{\frac{1}{c}}$$

7. (25 pts.) A remote stress $S = 50$ MPa is applied to the infinite plate shown below causing the plate to yield locally. The local stresses during the application of the stress S is given below. Calculate the residual stress field in the plate after the remote stress S is removed.



Elastic distribution near a hole:

$$\sigma = S \left[1 + 0.5 \left(\frac{R}{x} \right)^2 + 1.5 \left(\frac{R}{x} \right)^4 \right]$$

$$\sigma_A = 3 \cdot S = 150 \text{ MPa}$$

$$\sigma_B = S \left[1 + 0.5 \left(\frac{1}{3} \right)^2 + 1.5 \left(\frac{1}{3} \right)^4 \right] = 53.7 \text{ MPa}$$

$$\sigma_C \cong S = 50 \text{ MPa}$$

