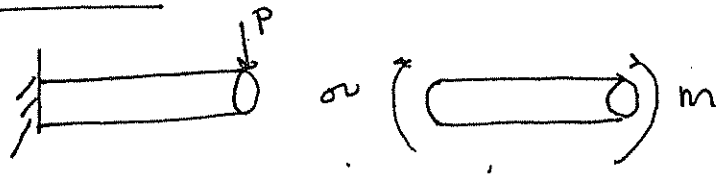
ME 461 – Fatigue and Fracture
Homework 2

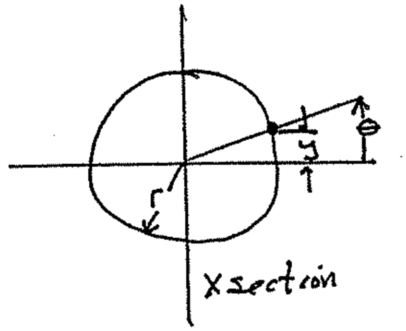
Due:
Wed., Feb. 13 (on-campus students)
Wed., Feb. 20 (outreach students)

1. Problem 4, page 91
2. Problem 5, page 91.
3. Problem 6, page 91.
4. Problem 10, page 92.
5. Problem 12, page 92.

(All problems are from the textbook)

Problem 4.4





$$S = my/I$$

$$= \frac{m}{I}(y) = \frac{m}{I}(r\sin\theta)$$

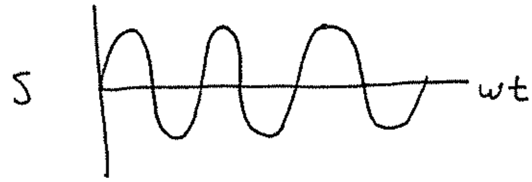
$$= \frac{m}{I} r \sin(\omega t)$$

which is sinusoidal

$\omega = \text{constant} = d\theta/dt$

$\theta = \int \omega\, dt + \phi_0$

$= \omega t$



Problem 4.5

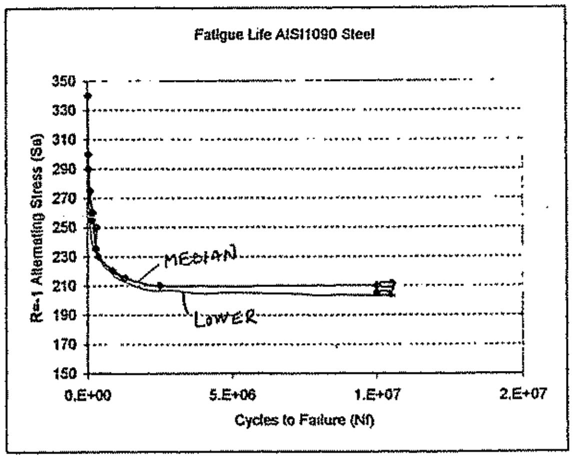
Adv/disadv.: The rectangular coordinate system condenses the data too much and does not provide a clear picture of the S-N behavior. The log-log plot extends the data and allows much better interpretation of the data. It gives more accurate estimates of S_f, S_{Nf} and N_f. From the log-log plot

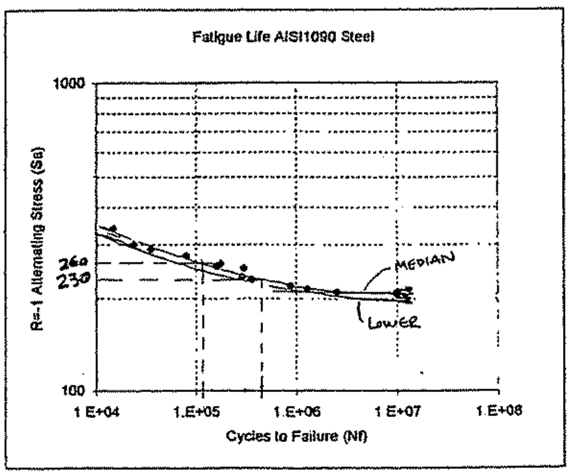
$S_f \approx$ 205 to 210 MPa, $S_{Nf}\big]_{5\times10^5} \approx$ 230 MPa

$N_f\big]_{S_a=260\,MPa} \approx 1.2\times10^5$ cycles

Scatter was most noticeable at longer lives. The above values were estimated by eye as reasonable values. Not enough data existed to justify more exact methods here.

Sa (Mpa)	Nf
340	1.50E+04
300	2.40E+04
290	3.60E+04
275	8.00E+04
260	1.77E+05
255	1.62E+05
250	3.01E+05
235	2.90E+05
230	3.61E+05
220	8.81E+05
215	1.30E+06
210	2.50E+06
210	1.00E+07
210	1.00E+07
205	1.00E+07
205	1.00E+07
205	1.00E+07





Problem 4.6

1. In bending, a stress gradient $d\sigma/dy$ exists in the cross section which means the average stress across a small near surface element in the cross section will be less than that for the axial case, where no stress gradient exists. This average stress across the small near surface element may be more important than the maximum stress. Since the bending average stress on the element is smaller than for the axial case, higher fatigue strengths are implied for the bending case.
2. Axial specimens may have greater misalignment problems than bending specimens and hence superimposed bending could exist in the axial tests that will increase stresses and hence provide lower fatigue strengths.
3. The probability of more microscopic discontinuities subjected to high stresses is larger in uniform axial loading compared to bending where only the outside fibers are subjected to higher stresses.
4. Since the entire axial loaded cross section is exposed to $S = P/A$ rather than $S = My/I$, greater hysteresis energy can be involved that could heat the axial specimen and lower the fatigue resistance.
5. Under bending, cracks will be growing into a lower nominal stress field than with axial loading which will provide lower fatigue crack growth rates and hence higher fatigue crack growth life.

Problem 4.10

$$S_a = \frac{S_{max} - S_{min}}{2} = \frac{S_{max} - (-35)}{2} = \frac{S_{max} + 35}{2}$$

$$S_m = \frac{S_{max} + S_{min}}{2} = \frac{S_{max} - 35}{2}$$

mod. Goodman:

$$S_a/S_f + S_m/S_u = 1 = \frac{S_{max} + 35}{2(220)} + \frac{S_{max} - 35}{2(500)} = 1$$

$\therefore$ $S_{max} = 290\text{ MPa}$

Morrow:

$$S_a/S_f + S_m/\sigma_f = 1 = \frac{S_{max} + 35}{2(220)} + \frac{S_{max} - 35}{2(600)} = 1$$

$\therefore$ $S_{max} = 304\text{ MPa}$

Difference ratio $= \frac{304}{290} = 1.05$ or 5% difference.

This difference is small and thus either model gives reasonable values and can be used interchangeably here.

Problem 4.12

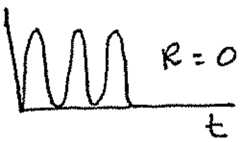
(a) Factors affecting S_f or S_{NF} include S_u, size and surface

Let $S_f \approx \frac{1}{2} S_u$ for small polished specimens

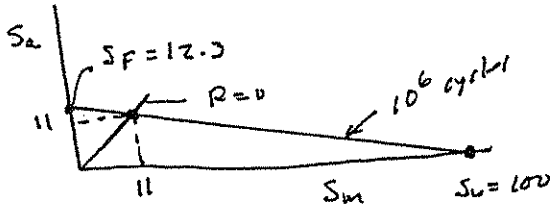
Let size effect factor, be $\sim$ 0.7 to 0.8

Let $K_S = 0.35$ from Fig 4.15 for as-forged

Thus $S_f\Big]_{10^6} \approx \left(\frac{100}{2}\right)(0.7)(.35) \approx \underline{12.3\ ksi}$

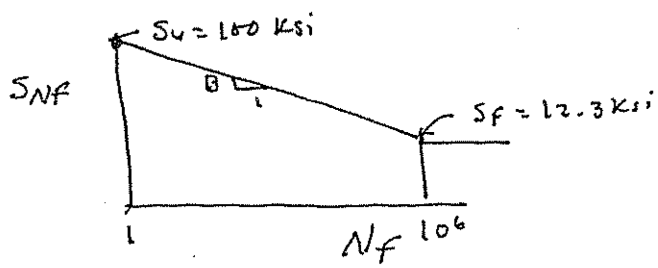
(b) 

$R = 0 \therefore S_a = S_m$



$$\frac{S_a}{S_f} + \frac{S_m}{S_u} = \frac{S_a}{12.3} + \frac{S_a}{100} = 1$$

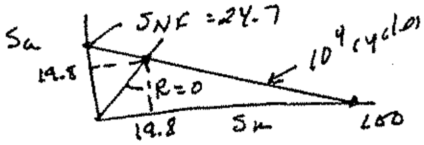
$\therefore \underline{S_a = 11 ksi} = \underline{S_m = 11 ksi}$ (10^6 cycles)

(c) 

$S_{NF} = A(N_F)^B$ & $A = S_u$

$$B = \frac{1}{6}\log\frac{S_f}{S_u} = \frac{1}{6}\log\left(\frac{12.3}{100}\right)$$

$= -0.152$



$S_{NF} = 100(10^4)^{-0.152} = 24.7 ksi$

$S_a/S_{NF} + S_m/S_u = \frac{S_a}{24.7} + \frac{S_a}{100} = 1 \therefore \underline{S_a = S_m = 19.8 ksi}$ (10^4 cycles)

check yielding $S_{max} = S_a + S_m \leq S_y$

$19.8 + 19.8 = 39.6 \leq 75$

$\therefore$ no yielding in a → c

Problem 4.12

(a) Factors affecting S_f or S_{Nf} include S_u, size and surface

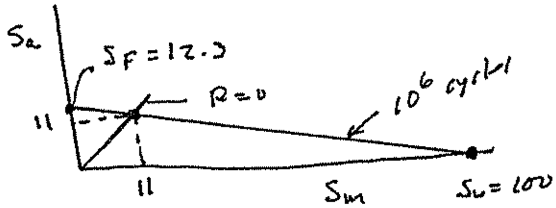
Let $S_f \approx 1/2\, S_u$ for small polished specimens

Let size effect factor, be ~ 0.7 to 0.8

Let $K_S = 0.35$ from Fig 4.15 for as-forged

Thus $S_f\Big]_{10^6} \approx \left(\frac{100}{2}\right)(0.7)(.35) \approx \underline{\underline{12.3\text{ ksi}}}$

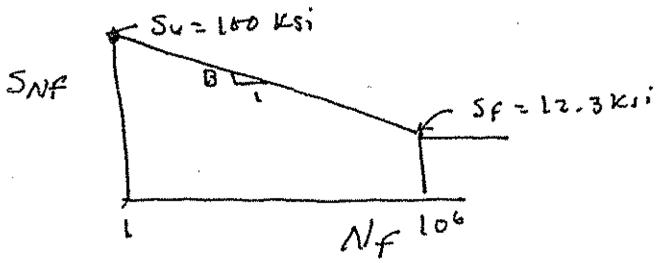
(b)  $R = 0 \therefore S_a = S_m$



$$\frac{S_a}{S_f} + \frac{S_m}{S_u} = \frac{S_a}{12.3} + \frac{S_a}{100} = 1$$

$\therefore \underline{\underline{S_a = 11\text{ksi}}} = \underline{\underline{S_m = 11\text{ksi}}}$ (10^6 cycles)

(c)



$S_{Nf} = A(N_f)^B$ & $A = S_u$

$$B = \frac{1}{6}\log\frac{S_f}{S_u} = \frac{1}{6}\log\left(\frac{12.3}{100}\right)$$

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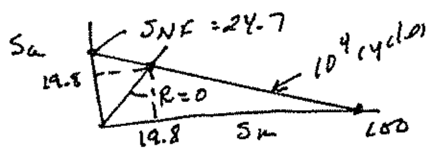

$$S_{Nf} = 100\left(10^4\right)^{-0.152} = 24.7\text{ksi}$$

$S_a/S_{Nf} + S_m/S_u = \frac{S_a}{24.7} + \frac{S_a}{100} = 1 \therefore \underline{\underline{S_a = S_m = 19.8\text{ksi}}}$ (10^4 cycles)

check yielding $S_{max} = S_a + S_m \le S_y$

$19.8 + 19.8 = 39.6 \le 75$

$\therefore$ no yielding in a $\to$ c