ME 461 – Fatigue and Fracture
Homework 3

Due:
Wed., Mar. 11 (on-campus students)
Wed., Mar. 25 (outreach students)

1. Problem 3, page 119
2. Problem 10, page 120.
3. Problem 1, page 180.
5. Problem 5, page 181.

(All problems are from the textbook)
Problem 6.4 cont.

For 0°C: \(2.5 \left(\frac{150}{500}\right)^2 = 22.5\ mm > b \) or \(a\)

For -50°C: \(2.5 \left(\frac{60}{550}\right)^2 = 3.0\ mm \leq b \) but \(a\)

Thus not in plane \(E\), if plane \(T\):

0°C: \(\gamma_y = \frac{1}{2\pi} \left(\frac{150}{500}\right)^2 = 14.3\ mm\) and \(2\gamma_y = 28.6\ mm\)

-50°C: \(\gamma_y = \frac{1}{2\pi} \left(\frac{60}{550}\right)^2 = 1.9\ mm\) and \(2\gamma_y = 3.8\ mm\)

For valid LEFM:

For 0°C: \(\gamma_y \leq a/8 = 10/8 = 1.25\ mm\)

For -50°C: \(\gamma_y \leq a/8 = 1.25\)

Since \(\gamma_y > 1.25\ mm\) in all cases: \(\boxed{\text{LEFM not valid}}\)

Significance is that LEFM will always give answers but must be checked to see if the model(s) are applicable, i.e.: 

- \(S \leq 0.8\ Sy\)
- \(\gamma_y \leq a/8, b/8, (w-a)/8\)
- For plane \(E\); \(b, a \geq 2.5(K_{ic}/Sy)^2\)
Problem 5.3

a) \( S = \frac{P}{A_0} = \frac{P}{\pi (d/2)^2} = 0.0321P \)  (P in N, S in MPa)

\( \varepsilon = \frac{\Delta l}{l_0} = \frac{\Delta l}{12.7} = 0.0787 \Delta l \)  (\( \Delta l \) in mm)

\[
\begin{align*}
\{ & \sigma = S (1 + \varepsilon) \text{ up to ultimate tensile strength, or} \\
& \varepsilon = \ln (1 + \varepsilon) \text{ initiation of necking}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Displacement, mm</th>
<th>Load, N</th>
<th>e</th>
<th>S, MPa</th>
<th>( \varepsilon )</th>
<th>( \sigma )</th>
<th>( \varepsilon_p )</th>
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<tbody>
<tr>
<td>0</td>
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<td>8600</td>
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<td>0.03150</td>
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<td>320.3</td>
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<td>0.09388</td>
<td>355.9</td>
<td>0.0887</td>
</tr>
<tr>
<td>1.8</td>
<td>9630</td>
<td>0.14173</td>
<td>308.9</td>
<td></td>
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</tr>
</tbody>
</table>

\( \sigma \) vs. \( \varepsilon \)

\( S \) vs. \( \varepsilon \)
b) \[ E = \frac{\Delta S}{\Delta e} = \frac{275}{0.0040} = 68750 \text{ MPa or 68.75 GPa} \]

\[ S_y = 293 \text{ MPa (see S-E curve)} \]

\[ S_u = 324 \text{ MPa (see S-E curve)} \]

\[ Q_f = \frac{P_f/A_f}{(1+4R/D_{min}) \ln (1+D_{min}/4R)} = \frac{7200/(\frac{11}{4} (4.2)^2)}{[1+4(3.3)/4.2] \ln [1+4.2/4(3.3)]} \]

\[ = \frac{519.69}{(4.143)(0.276)} = 454 \text{ MPa} \]

\[ \varepsilon_f = \ln \left( \frac{A_0}{A_f} \right) = \ln \left( \frac{D_0}{D_{min}} \right)^2 = \ln \left( \frac{13}{4.2} \right)^2 = 0.81 \text{ or 81%} \]

\[ \% RA = \frac{A_0 - A_f}{A_0} \times 100 = \frac{D_0}{D_{min}} - 1 \times 100 = \frac{(6.3)^2 - (4.2)^2}{(6.3)^2} \times 100 = 56\% \]

C) Choose several points approximately equally spaced between yielding and ultimate tensile strength on the true stress-strain curve (typically between 5 to 10 points) and plot \( \varepsilon_p = \varepsilon - \frac{\sigma}{E} \) versus \( \sigma \) on a log-log plot. The best fit line gives \( k = 410 \text{ MPa} \) and \( n = 0.067 \). See graph.
$K = 410 \text{ MPa}$

$n = 0.067$
Problem 5.10

\[ \frac{\Delta \varepsilon_e}{2} = \frac{\varepsilon_f'}{E} (2N_f)^{1/2} = \frac{104.3}{2 \cdot 10^6} (2N_f)^{-0.107} \]

\[ \frac{\Delta \varepsilon_p}{2} = \varepsilon_f' (2N_f)^{3/4} = 0.309 (2N_f)^{-0.481} \]

\[ \varepsilon_a = \frac{104.3}{2 \cdot 10^6} (2N_f)^{-0.107} + 0.309 (2N_f)^{-0.481} \]

\[ 2N_f = \left( \frac{\varepsilon_f'}{\sigma_f'} \right)^{b-c} = \left( \frac{0.309 \times 2 \cdot 10^6}{104.3} \right)^{-0.107+0.481} = 55,674 \]

or \[ N_f = 27,836 \text{ cycles} \]

\[ \varepsilon_a = \frac{104.3}{2 \cdot 10^6} \left( 55,674 \right)^{-0.107} + 0.309 \left( 55,674 \right)^{-0.481} \]

\[ = 0.00161 + 0.00161 = 0.00322 \]
Problem 6.1

Using the principle of superposition, \( K_I = K_{ISEN} + K_{IBEND} \)

Using Fig. 6.3(b) for \( K_{ISEN} \) with \( a/w = 6/120 = 0.05 \)

\[ Y = 2.05, \text{ thus } K_{ISEN} = \frac{YPa^{1/2}}{BW} = \frac{(2.05)(100 \times 0.03)(0.06)^{1/2}}{(0.01)(0.120)} \]

\[ K_{ISEN} = 13.23 \times 10^6 \text{ Pa} \sqrt{\text{m}} = 13.23 \text{ MPa} \sqrt{\text{m}} \]

Using Fig. 6.3(d) for \( K_{IBEND} \) (pore bonding) with \( a/w = 0.05 \)

\[ Y = 1.9, \text{ thus } K_{IBEND} = \frac{Y6Ma^{1/2}}{BW^2} = \frac{(1.9)(6)(100 \times 0.03)(0.04)(0.06)^{1/2}}{(0.01)(0.120)^2} \]

\[ K_{IBEND} = 24.53 \times 10^6 \text{ Pa} \sqrt{\text{m}} = 24.53 \text{ MPa} \sqrt{\text{m}} \]

Thus \( K_I = (13.23 + 24.53) \text{ MPa} \sqrt{\text{m}} = 37.76 \text{ MPa} \sqrt{\text{m}} \approx 38 \text{ MPa} \sqrt{\text{m}} \)

For \( a = 30 \text{ mm} \), \( a/w = 30/120 = 0.25 \)

For \( K_{ISEN} \), \( Y = 2.7 \) and for \( K_{IBEND} \), \( Y = 1.95 \)

\[ K_I = K_{ISEN} + K_{IBEND} = \frac{(2.7)(100 \times 0.03)(0.03)^{1/2}}{(0.01)(0.120)} + \frac{(1.95)(6)(4000)(0.03)^{1/2}}{(0.01)(0.120)^2} \]

\[ K_I = 38.97 \text{ MPa} \sqrt{\text{m}} + 56.29 \text{ MPa} \sqrt{\text{m}} = 95.26 \text{ MPa} \sqrt{\text{m}} = 95 \text{ MPa} \sqrt{\text{m}} \]
Problem 6.4

Given A553B steel from Fig. 6.9

At 0°C, \( K_{ic} = 150 \text{MPa}\sqrt{m} \) and \( S_y = 500 \text{MPa} \)
At -50°C, \( K_{ic} = 60 \text{MPa}\sqrt{m} \) and \( S_y = 550 \text{MPa} \)

Force for yielding with no crack; \( S = P/A_o \); \( P_{yield} = S_y A_o \)

0°C: \( P_{yield} = (500 \text{MPa})(0.05)(0.1) = 2.5 \text{MN} \)

-50°C: \( P_{yield} = (550 \text{MPa})(0.05)(0.1) = 2.75 \text{MN} \)

Force for fracture with a crack;

For \( a/c = 1 \), \( \phi = \pi/2 \); \( K_I = \frac{(1.12)^2 S \sqrt{\pi a}}{\phi} \)

0°C: \( P_f = (150 \times 10^6)(0.05)(0.1)/1.41(0.01)^{1/2} = 5.32 \text{MN} \)

-50°C: \( P_f = (60 \times 10^6)(0.05)(0.1)/1.41(0.01)^{1/2} = 2.13 \text{MN} \)

At 0°C, the bar will yield first; at -50°C, the bar will fracture first. Use of LEFM is good at 0°C because of general yielding. At -50°C, LEFM may be applicable at -50°C, but must check other criteria.

For valid LEFM; \( \gamma \leq 9/8 \), \( B/q, (L-q)/q \). If use LEFM for plane \( \varepsilon \); \( B/q \geq 2.5(K_{ic}/S_y)^2 \)
Problem 6.5
Given annealed Ti-6Al-4V; \( K_{IC} = 85 \text{ MPa} \cdot \sqrt{\text{m}}, S_y = 815 \text{ MPa} \)

For plane \( T \) or plane \( E \): 
\[ 2.5 \left( \frac{85}{815} \right)^2 = 27.2 \text{ mm} \]
Since \( B = 25 \text{ mm} < 27.2 \text{ mm} \), we have \( \text{plane } T \) but close to plane \( E \).
For \( a/c = 1 \); \( \phi = \frac{\pi}{2} \). For \( a = 8 \text{ mm} \),

\[ K_I = \frac{1.12 S \sqrt{\pi(0.08)}}{\sqrt{\text{sec}(\pi(8)/2(25))}} = 0.1208 S \left( \text{Pa} \cdot \sqrt{\text{m}} \right) \]

For \( K_e = 105 \text{ MPa} \cdot \sqrt{\text{m}} \); \( S_{\text{free}} = 105 \times 10^6/1208 = 86.9 \text{ MPa} \)
Since \( S_{\text{free}} > S_y \), component will yield first; also
**LEFM not valid** since \( S_{\text{free}} > 0.8 S_y \).

If thickness doubled; \( B = 50 \text{ mm} \); this satisfies thickness requirement for plane \( E \), but crack length (8mm) still less than general requirement for plane \( E \). If we let thickness requirement control stress state, then let us assume plane \( E \). If so, we can determine fracture stress. Thus,

For \( B = 50 \text{ mm} \); \( K_I = \frac{1.12 S \sqrt{\pi(0.08)}}{\sqrt{\text{sec}(\pi(8)/2(50))}} \)

\[ K_I = 0.115 S \left( \text{Pa} \cdot \sqrt{\text{m}} \right) \]; for Fracture;

\[ S_{\text{frac}} = K_{IC}/0.115 = 85 \times 10^6/0.115 = 739 \text{ MPa} \]
Check LEFM; \( S \leq 0.8 S_y = 0.8(815) = 652 \text{ MPa} \)

Since \( S_{\text{frac}} > 0.8 S_y \) **LEFM not valid** for either thickness.
Problem 6.6

From Table A.3:

- 200 grade: $S_y = 1450\text{MPa}$; $K_{ic} = 110\text{MPa}\sqrt{\text{m}}$
- 300 grade: $S_y = 1905\text{MPa}$; $K_{ic} = 50-64\text{MPa}\sqrt{\text{m}}$

If no cracks, use $S_y$ since $S_u$ not given

\[
\frac{P_{allow}(300)}{P_{allow}(200)} = \frac{S_y(300)A}{S_y(200)A} = \frac{1905}{1450} = 1.34 \text{ or } 34\% \text{ increase in load based on yielding}
\]

If a crack present:
\[K_{ic} = S\sqrt{\pi a}\]

For fracture with a crack, use $K_{ic} = K_{ic}$, thus

\[\left(\frac{P}{A}\right)\sqrt{\pi a} = K_{ic} ;\]

\[
\frac{P_{frac}(300)}{P_{frac}(200)} = \frac{K_{ic}(300)A\sqrt{\pi a}}{K_{ic}(200)A\sqrt{\pi a}} = \frac{50-64}{110} \approx \frac{1}{2} \text{ or a load drop of } \approx 50\% \]

If this is a monotonic load failure then:
- use NDI inspection and replace cracked parts.
- determine cause of failure
- use additional members or different members
- reduce load
- verify stress state condition (plane $\varepsilon$)
- use a tougher material