ME 461 – Fatigue and Fracture
Homework 3

Due:
Wed., Mar. 11 (on-campus students)
Wed., Mar. 25 (outreach students)

1. Problem 3, page 119
2. Problem 10, page 120.
3. Problem 1, page 180.
4. Problem 4, page 181.
5. Problem 5, page 181.
6. Problem 6, page 181.

(All problems are from the textbook)

Problem 6.4 cont.

For $0°C$; $2.5(150/500)^2 = 225\,mm \geq B$ or a

For $-50°C$; $2.5(60/550)^2 \approx 30\,mm \leq B$ but $\geq a$

Thus not in plane ϵ. If plane σ;

$0°C$; $r_y = \frac{1}{2\pi}\left(\frac{150}{500}\right)^2 = 14.3\,mm$ and $2r_y = 28.6\,mm$

$-50°C$: $r_y = \frac{1}{2\pi}\left(\frac{60}{550}\right)^2 = 1.9\,mm$ and $2r_y = 3.8\,mm$

For valid LEFM;

For $0°C$: $r_y \leq a/8 = 10/8 = 1.25\,mm$

For $-50°C$; $r_y \leq a/8 = 1.25$

Since $r_y > 1.25\,mm$ in all cases : $\boxed{\text{LEFM \underline{not} valid}}$

Significance is that LEFM will always give answers but must be checked to see if the model(s) are applicable, i.e.:

- $S \leq 0.8 S_y$
- $r_y \leq a/8,\ B/8,\ (w-a)/8$
- For plane ϵ; $B, a \geq 2.5(K_{Ic}/S_y)^2$

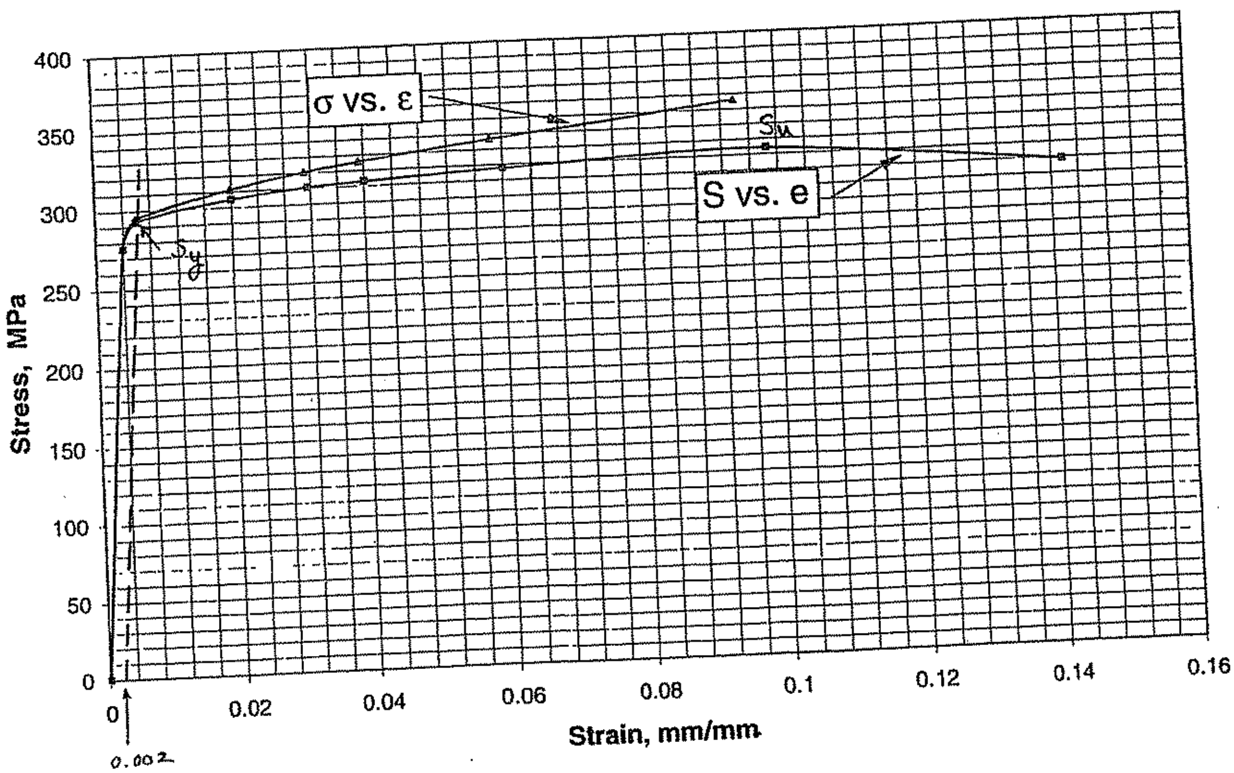
Problem 5.3

a) $S = \frac{P}{A_0} = \frac{P}{\frac{\pi}{4}(6.3)^2} = 0.0321 P$ (P in N, S in MPa)

$e = \frac{\Delta l}{l_0} = \frac{\Delta l}{12.7} = 0.0787\,\Delta l$ (Δl in mm)

$$\begin{cases} \sigma = S(1+e) \\ \epsilon = \ln(1+e) \end{cases}$$ up to ultimate tensile strength, or initiation of necking

Displacement, mm	Load, N	e	S, MPa	ε	σ	ε_p
0	0	0	0	0	0	0
0.05	8600	0.00394	275.9	0.00393	277.0	
0.075	9130	0.00591	292.9	0.00589	294.6	
0.1	9360	0.02000	305.0	0.01980	311.1	0.0153
0.4	9680	0.03150	310.5	0.03101	320.3	0.0264
1	10000	0.04000	313.0	0.03922	325.5	0.0345
		0.06000	317.5	0.05827	336.6	0.0534
1.25	10100	0.09843	324.0	0.09388	355.9	0.0887
1.8	9630	0.14173	308.9			



b) $E = \frac{\Delta S}{\Delta e} = \frac{275}{0.0040} = 68{,}750$ MPa or 68.75 GPa

$S_y = 293$ Mpa (see S-e curve)

$S_u = 324$ Mpa (see S-e curve)

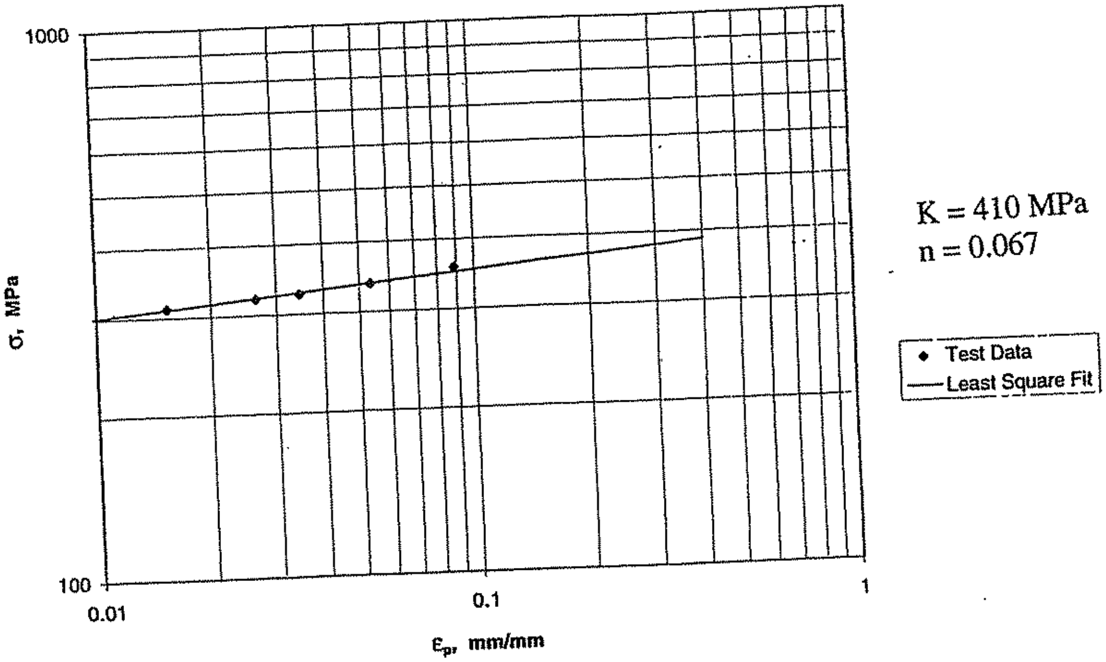
$$\sigma_f = \frac{P_f/A_f}{(1+4R/D_{min})\ln(1+D_{min}/4R)} = \frac{7200/[\frac{\pi}{4}(4.2)^2]}{[1+4(3.3)/4.2]\ln[1+4.2/4(3.3)]}$$

$$= \frac{519.69}{(4.143)(0.276)} = \underline{454 \text{ MPa}}$$

$$\epsilon_f = \ln\left(\frac{A_o}{A_f}\right) = \ln\left(\frac{D_o}{D_{min}}\right)^2 = \ln\left(\frac{6.3}{4.2}\right)^2 = 0.81 \text{ or } 81\%$$

$$\%\,RA = \frac{A_o - A_f}{A_o} \times 100 = \frac{D_o^2 - D_{min}^2}{D_o^2} \times 100 = \frac{(6.3)^2 - (4.2)^2}{(6.3)^2} \times 100 = 56\%$$

c) Choose several points approximately equally spaced between yielding and ultimate tensile strength on the true stress - strain curve (typically between 5 to 10 points) and plot $\epsilon_p = \epsilon - \sigma/E$ versus σ on a log-log plot. The best fit line gives $k = 410$ MPa and $n = 0.067$. See graph.



1000
100
0.01
0.1
1
σ, MPa
ε_p, mm/mm
K = 410 MPa
n = 0.067
Test Data
Least Square Fit

problem 5.10

$$\frac{\Delta \epsilon_e}{2} = \frac{\sigma_f'}{E}(2N_f)^b = \frac{1043}{201000}(2N_f)^{-0.107}$$

$$\frac{\Delta \epsilon_p}{2} = \epsilon_f'(2N_f)^c = 0.309(2N_f)^{-0.481}$$

$$\epsilon_a = \frac{1043}{201000}(2N_f)^{-0.107} + 0.309(2N_f)^{-0.481}$$

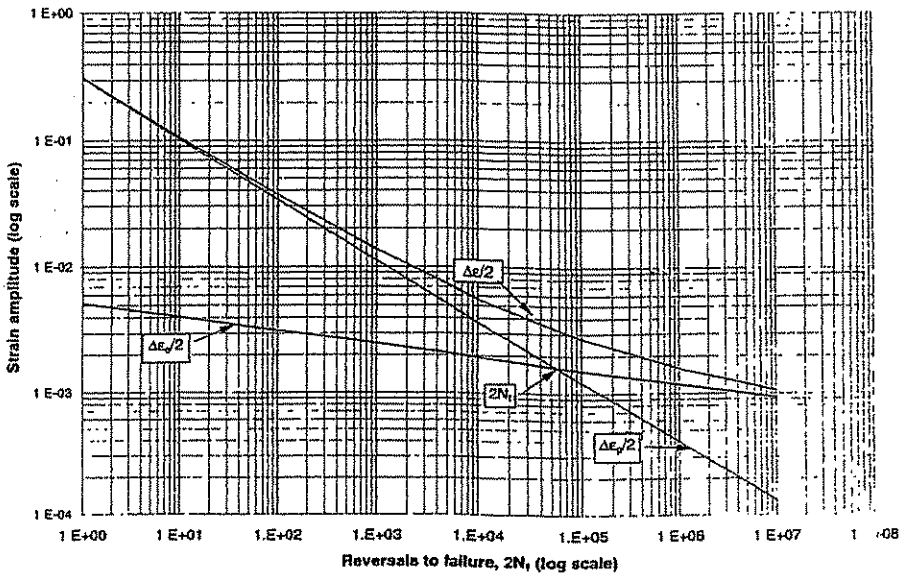
$$2N_t = \left(\frac{\epsilon_f' E}{\sigma_f'}\right)^{\frac{1}{b-c}} = \left(\frac{0.309 \times 201000}{1043}\right)^{\frac{1}{-0.107+0.481}}$$

$$= 55,674$$

$$\text{or} \quad N_t = 27,836 \text{ cycles}$$

$$\epsilon_a = \frac{1043}{201000}(55674)^{-0.107} + 0.309(55674)^{-0.481}$$

$$= 0.00161 + 0.00161 = 0.00322$$



Problem 6.1

Using the principle of superposition; $K_I = K_{ISEN} + K_{IBEND}$

Using Fig. 6.3(b) for K_{ISEN} with $a/w = 6/120 = .05$

$Y \approx 2.05$, thus $K_{I\,SEN} = \dfrac{Y P a^{1/2}}{BW} = \dfrac{(2.05)(100\times10^3)(.006)^{1/2}}{(.01)(0.120)}$

$K_{I\,SEN} = 13.23\times10^6\ Pa\sqrt{m} = 13.23\,MPa\sqrt{m}$

Using Fig. 6.3(d) for $K_{I\,BEND}$ (pure bending) with $a/w = 0.05$

$Y \approx 1.9$, thus $K_{I\,BEND} = \dfrac{Y 6 M a^{1/2}}{B w^2} = \dfrac{(1.9)(6)(100\times10^3 \cdot 0.04)(.006)^{1/2}}{(.01)(0.120)^2}$

$K_{I\,BEND} = 24.53\times10^6\ Pa\sqrt{m} = 24.53\,MPa\sqrt{m}$

Thus $K_I = (13.23 + 24.53)\ MPa\sqrt{m} = \boxed{37.76\,MPa\sqrt{m} \approx 38\,MPa\sqrt{m}}$

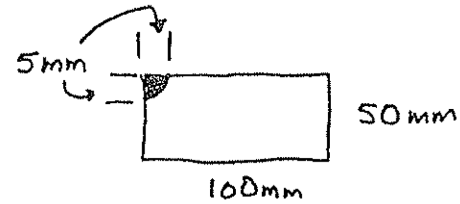
For $a = 30mm$; $a/w = 30/120 = 0.25$

For K_{ISEN}, $Y \approx 2.7$ and for $K_{I\,BEND}$, $Y \approx 1.95$

$K_I = K_{ISEN} + K_{I\,BEND} = \dfrac{(2.7)(100\times10^3)(.03)^{1/2}}{(.01)(0.120)} + \dfrac{(1.95)(6)(4000)(.03)^{1/2}}{(.01)(.12)^2}$

$K_I = 38.97\,MPa\sqrt{m} + 56.29\,MPa\sqrt{m} = \boxed{95.26\,MPa\sqrt{m} \approx 95\,MPa\sqrt{m}}$

Problem 6.4



Given A553B steel from Fig. 6.9

At 0°C, $K_{Ic} \approx 150\,MPa\sqrt{m}$ and $S_y = 500\,MPa$
At -50°C, $K_{Ic} = 60\,MPa\sqrt{m}$ and $S_y \approx 550\,MPa$

Force for yielding with no crack; $S = P/A_o$; $P_{yield} = S_y A_o$

0°C: $P_{yield} = (500\,MPa)(.05)(.1) \boxed{= 2.5\,MN}$

-50°C; $P_{yield} = (550\,MPa)(.05)(.1) \boxed{= 2.75\,MN}$

Force for fracture with a crack;

For $a/c = 1$, $\phi = \pi/2$; $K_I = \dfrac{(1.12)^2 S\sqrt{\pi a}}{\phi} = 1.41 \dfrac{P}{A_o}\sqrt{a}$

0°C: $P_f = (150 \times 10^6)(.05)(.1)/1.41(.01)^{1/2} \boxed{= 5.32\,MN}$

-50°C: $P_f = (60 \times 10^6)(.05)(.1)/1.41(.01)^{1/2} \boxed{= 2.13\,MN}$

At 0°C, the bar will yield first, at -50°C, the bar will fracture first. Use of LEFM is no good at 0°C because of general yielding. At -50°C, LEFM may be applicable at -50°C, but must check other criteria.

For valid LEFM; $r_y \leq a/8, B/8, (w-a)/8$. If use LEFM

for plane ϵ; $B, a \geq 2.5(K_{Ic}/S_y)^2$

Problem 6.5

Given annealed Ti-6Al-4V; $K_{Ic} = 85 MPa\sqrt{m}$, $S_y = 815 MPa$

For plane σ or plane ϵ: $2.5(85/815)^2 = 27.2 mm$
Since $B = 25mm < 27.2mm$, we have $\boxed{\text{plane } \sigma}$, but close to plane ϵ.
For $a/c = 1$; $\phi = \frac{\pi}{2}$. For $a = 8mm$,

$$K_I = \frac{1.12 S\sqrt{\pi(.008)}}{\pi/2}\sqrt{\sec(\pi(8)/2(25))} = 0.1208 S \; (Pa\sqrt{m})$$

For $K_c = 105 MPa\sqrt{m}$; $S_{frac} = 105 \times 10^6 / .1208 = 869 MPa$
since $S_{frac} > S_y$, component will yield first; also $\boxed{\text{LEFM not valid}}$ since $S_{frac} > .8 S_y$.

If thickness doubled; $B = 50mm$; this satisfies thickness requirement for plane ϵ, but crack length (8mm) still less than general requirement for plane ϵ. If we let thickness requirement control stress state, then let us assume plane ϵ. If so, we can determine fracture stress. Thus,

For $B = 50 mm$; $$K_I = \frac{1.12 S\sqrt{\pi(.008)}}{\pi/2}\sqrt{\sec \pi(8)/2(50)}$$

$K_I = 0.115 S \; (Pa\sqrt{m})$; For Fracture;

$$S_{frac} = K_{Ic}/0.115 = 85 \times 10^6 / 0.115 = 739 MPa$$

Check LEFM; $S \le 0.8 S_y = .8(815) = 652 MPa$

Since $S_{frac} > .8 S_y$ $\boxed{\text{LEFM not valid}}$ for either thickness.

Problem 6.6

From Table A.3 : 200 grade $S_y = 1450\,MPa$; $K_{Ic} = 110\,MPa\sqrt{m}$

300 grade $S_y = 1905\,MPa$; $K_{Ic} = 50\text{-}64\,MPa\sqrt{m}$

If no cracks, use S_y since S_u not given

$$\frac{P_{allow}(300)}{P_{allow}(200)} = \frac{S_y(300)\cancel{A}}{S_y(200)\cancel{A}} = \frac{1905}{1450} = \boxed{1.34 \text{ or } 34\% \text{ increase in load based on yielding}}$$

If a crack present ; $K_I = S\sqrt{\pi a}\,\alpha$

For fracture with a crack, use $K_I = K_{Ic}$, thus

$(P/A)\sqrt{\pi a}\,\alpha = K_{Ic}$;

$$\frac{P_{frac}(300)}{P_{frac}(200)} = \frac{K_{Ic}(300)\cancel{A\sqrt{\pi a}\,\alpha}}{K_{Ic}(200)\cancel{A\sqrt{\pi a}\,\alpha}} = \frac{50-64}{110} \boxed{\approx \frac{1}{2} \text{ or a load drop of } \approx 50\%}$$

If this is a monotonic load failure then :

- use NDI inspection and replace cracked parts.
- determine cause of failure
- Use additional members or different members
- reduce load
- verify stress state condition (plane ϵ)
- Use a tougher material