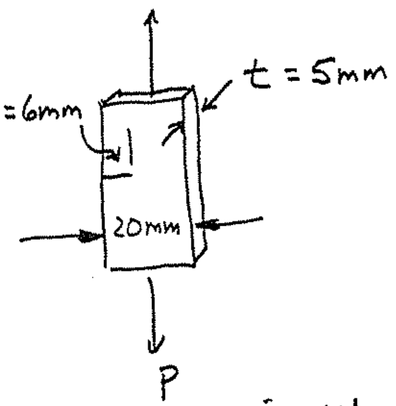
ME 461 – Fatigue and Fracture
Homework 4

Due:
Fri., Mar. 27 (on-campus students)
Fri., Apr. 10 (outreach students)

1. Problem 10, page 182.
2. Problem 11, page 182.
3. Problem 13, page 183.
4. Problem 16, page 184.
5. Problem 17, page 184.

(All problems are from the textbook)

Problem 6.10



Given: $A = 10^{-12}$ and $n = 3.5$

$R = 0$

Assume Paris behavior

$a/w = 6/20 = 0.3 \qquad \Delta K = \dfrac{Y \Delta P \sqrt{a}}{tW}$

in MN

$Y = 2.95; \quad \Delta K = \dfrac{2.95(P_{max} - 0)\sqrt{.006}}{(.005)(.020)} = 2{,}285 P_{max} (MPa\sqrt{m})$

From Paris behavior; $\dfrac{da}{dN} \approx \dfrac{\Delta a}{\Delta N} = A(\Delta K)^n = 10^{-12}(2{,}285 P_{max})^{3.5}$

From Fig. 3.15(a) $\Rightarrow \Delta a \approx 6.2\,\mu m = 6.2 \times 10^{-6} m$

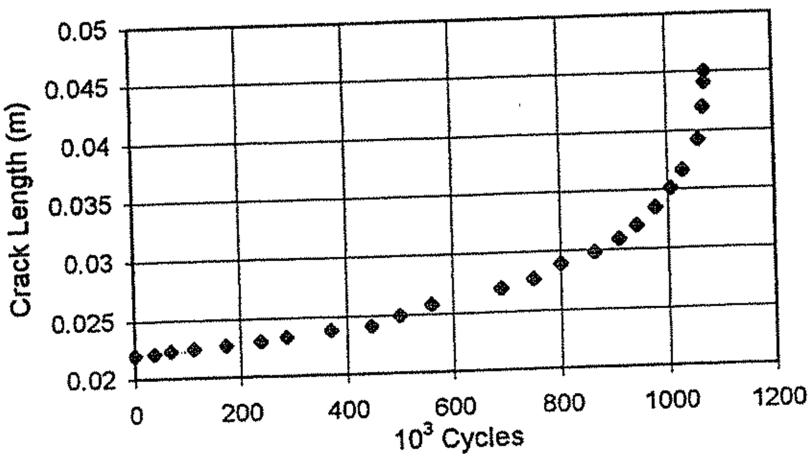
$\Rightarrow \Delta N \approx 26$ striations

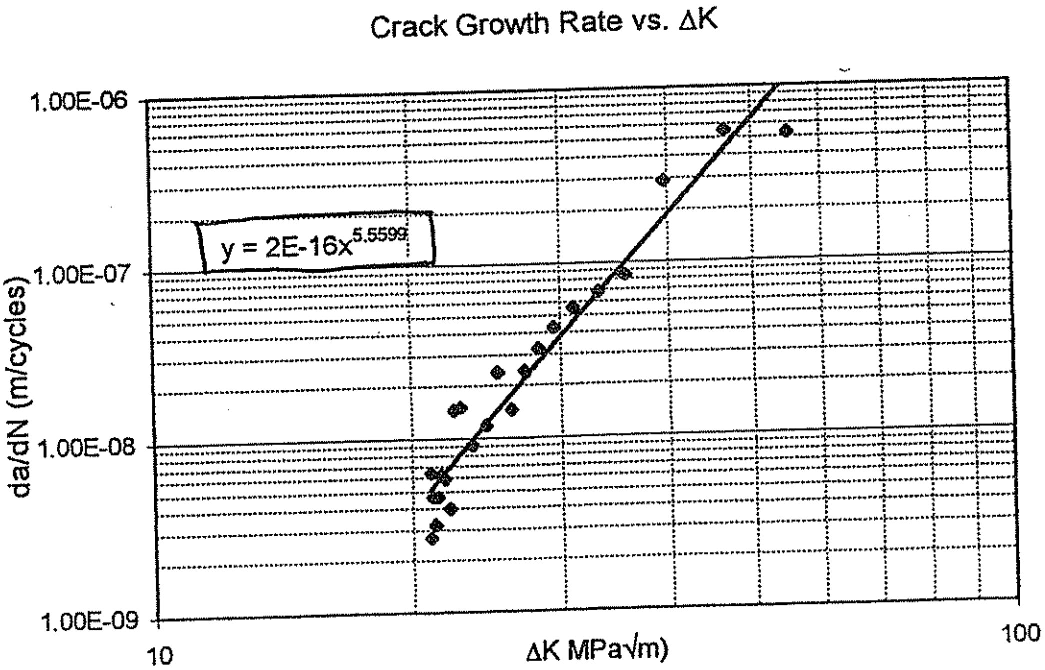
$\dfrac{\Delta a}{\Delta N} \approx \dfrac{6.2 \times 10^{-6} m}{26 \text{ striations}} = 2.4 \times 10^{-7} m/\text{cycle} = 10^{-12}(2{,}285 P_{max})^{3.5}$

$P_{max} = \left[\left(2.4 \times 10^{-7} / 10^{-12}\right)^{1/3.5}\right] / 2{,}285 = 0.0151$ MN

$\boxed{P_{max} = 15.1\,KN \quad \Rightarrow \quad \Delta K = 34.4\, MPa\sqrt{m}}$

Problem 6.11





$$\boxed{\begin{aligned} A &= 2\times10^{-16} \\ n &= 5.5599 \approx 5.56 \end{aligned}}$$

Problem 6.13

Low carbon steel: $S_y = 350MPa$, $S_u = 550MPa$, $K_c = 110\ MPa\sqrt{m}$
$\Delta K_{th}]_{R=0} = 5MPa\sqrt{m}$, $A = 6.9\times10^{-12}$, $n = 3$

$2a_i = 5mm \Rightarrow a_i = 2.5mm = 0.0025m$

(a) $K_I = S_{max}\sqrt{\pi a}$; $K_I]_A = (150)\sqrt{\pi(.0025)} = \boxed{13.3MPa\sqrt{m}}$

$13.3MPa\sqrt{m} > \Delta K_{th}]_{R=0}$ and $< K_c$: Crack will grow

(b) $\Delta K_1]_A = S_{max}\sqrt{\pi(.0025)} = \Delta K_{th} = 5MPa\sqrt{m}$; $\boxed{S_{max} = 56.4MPa}$

This is approximately $56.4/350 = 15\%$ of S_y. (LEFM OK)

(c) Crack length at B

$$N = 70{,}000 = \frac{2}{(3-2)(6.9\times10^{-12})(150)^3(\pi)^{3/2}}\left[\frac{1}{(.0025)^{.5}} - \frac{1}{(a_B)^{.5}}\right]$$

$a_B = 0.00418\ m = 4.18mm$; $\boxed{2a_B = 8.36mm}$

crack length at C

use $a_i = a_B = 4.18mm = 0.00418m$ and $\Delta S = 200MPa$

$$\frac{1}{a_c^{.5}} = -19.2\times10^{-12}(30{,}000)(200)^3 + \frac{1}{(0.00418)^{.5}}$$

$a_c = 0.00848m = 8.48m$; $\boxed{2a_c = 16.96mm}$

Problem 6.13 continued

Crack length at D

use $a_i = a_c = 0.00848\,m$ and $\Delta S = 250\,MPa$

$$\frac{1}{a_D^{.5}} = -19.2\times10^{-12}(10{,}000)(250)^3 + \frac{1}{(.00848)^{.5}}$$

$a_D = 0.0162\,m = 16.2\,mm$; $\boxed{2a_D = 32.4\,mm}$

(d) $K_{I\,B^-} = S\sqrt{\pi a} = 150\sqrt{\pi(.00418)} = 17.2\,MPa\sqrt{m}$ *

$K_{I\,B^+} = 200\sqrt{\pi(.00418)} = 22.9\,MPa\sqrt{m}$

$K_{I\,C^-} = 200\sqrt{\pi(.00848)} = 32.6\,MPa\sqrt{m}$ *

$K_{I\,C^+} = 250\sqrt{\pi(.00848)} = 40.8\,MPa\sqrt{m}$

$K_{I\,D} = 250\sqrt{\pi(0.0162)} = 56.4\,MPa\sqrt{m}$

* B^-, C^- is before load increase, B^+, C^+ is after load increase

(e) For plane stress $2r_y = \frac{1}{\pi}\left(K/S_y\right)^2$

at A: $2r_y = \frac{1}{\pi}\left(13.3/350\right)^2 = 0.00046\,m = \boxed{0.46\,mm}$

at B^-: $2r_y = \frac{1}{\pi}\left(17.2/350\right)^2 = 0.00077\,m = \boxed{0.77\,mm}$

at B^+: $2r_y = \frac{1}{\pi}\left(22.9/350\right)^2 = 0.00136\,m = \boxed{1.36\,mm}$

at C^-: $2r_y = \frac{1}{\pi}\left(32.6/350\right)^2 = 0.00276\,m = \boxed{2.76\,mm}$

at C^+: $2r_y = \frac{1}{\pi}\left(40.8/350\right)^2 = 0.00433\,m = \boxed{4.33\,mm}$

Problem 6.13 continued

(f) Since $K_{I_D} = 56.4 \text{ MPa}\sqrt{m} < K_c = 110 \text{ MPa}\sqrt{m}$, crack growth occurs beyond time D.

$$a_f = (K_c/S)^2/\pi = (110/250)^2/\pi = 0.0616\,m = 61.6\,mm$$

$$N_f = \frac{2}{(3-2)(6.9\times10^{-12})(250)^3(\pi)^{3/2}}\left[\frac{1}{(.0162)^{.5}} - \frac{1}{(.0616)^{.5}}\right]$$

$$= 12\,752 \text{ cycles} \approx 12\,750 \text{ cycles}$$

$$N_{TOTAL} = 70\,000 + 30\,000 + 10\,000 + 12\,750$$

$$\boxed{N_{TOTAL} = 122\,750 \approx 123\,000 \text{ cycles}}$$

(g) $S \leq 0.8 S_y$ for all stress levels.

From (e) earlier,

$r_y(A) = 0.23\,mm$

$r_y(B^+) = 0.68\,mm$

$r_y(C^+) = 2.16\,mm$

$r_y(D) = 4.13\,mm$

For cyclic conditions, $r_y \leq \frac{1}{4}a$ for valid LEFM.

For all cases, this is satisfied. At fracture, $r_y = (110/350)^2/2\pi = 0.0157\,m = 15.7\,mm$, where $a_f = 61.6\,mm$. Thus, $r_y \approx \frac{1}{4}a$ at fracture. Therefore, LEFM is very realistic.

Problem 6.16

For 7075-T6, $A = 2.7 \times 10^{-11}$, $n = 3.7$

For $R=0$, $da/dN = 2.7 \times 10^{-11} (\Delta K)^{3.7}$

(a)
using Walker equation and $\lambda = 0.5$,

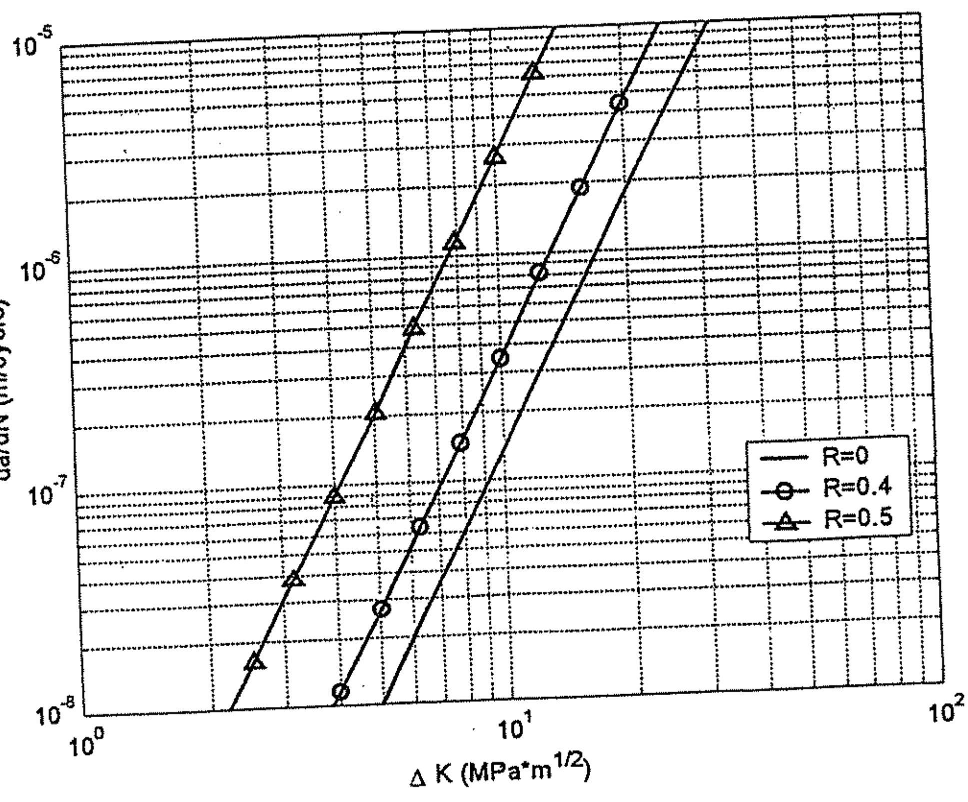
$\underline{R=0.4}$: $A'' = 2.7 \times 10^{-11} / (1-0.4)^{3.7(1-.5)} = 6.95 \times 10^{-11}$

$$\boxed{da/dN \Big]_{R=0.4} = 6.95 \times 10^{-11} (\Delta K)^{3.7}}$$

$\underline{R=0.8}$: $A'' = 2.7 \times 10^{-11} / (1-.8)^{3.7(1-.5)} = 5.3 \times 10^{-10}$

$$\boxed{da/dN \Big]_{R=0.8} = 5.3 \times 10^{-10} (\Delta K)^{3.7}}$$

(b) Reasonable region II between ~ $10^{-8} - 10^{-5}$ m/cycle



Problem 6.16 continued

(c) For $R=0$ to $R=0.4$; $\frac{da/dN]_{R=0.4}}{da/dN]_{R=0}} = \frac{A''\,]_{R=0.4}}{A\,]_{R=0}}$

$$\frac{A''\,]_{R=0.4}}{A\,]_{R=0}} = \frac{6.95\times10^{-11}}{2.7\times10^{-11}} \boxed{\approx 2.6 \text{ or } 2.6 \text{ times faster}}$$

For $R=0$ to $R=0.8$

$$\frac{A''\,]_{R=0.8}}{A\,]_{R=0}} = \frac{5.3\times10^{-10}}{2.7\times10^{-11}} \boxed{\approx 19.6 \text{ or } 19.6 \text{ times faster}}$$

The 19.6 times faster for $R=0.8$ seems very high. $\lambda \approx 0.5$ may not be a good estimate for this material.

Problem 6.17

$S_y = 350 MPa$, $S_u = 550 MPa$, $K_c = 110 MPa\sqrt{m}$

$\Delta K_{th}\big]_{R=0} = 5 MPa\sqrt{m}$, $A = 6.9\times10^{-12}$, $n = 3$

Given : $S_{min} = 100 MPa$ and $\lambda = 0.5$ and $\gamma = 0.3$

(a) $K_{I(max\text{-}A)} = S_{max}\sqrt{\pi a} = 150\sqrt{\pi(.0025)} = 13.3 MPa\sqrt{m}$ ($< K_c$ thus won't fracture)

$\Delta K_{I(A)} = \Delta S\sqrt{\pi a} = (150-100)\sqrt{\pi(.0025)} = 4.4 MPa$

From EQ 6.23, $\Delta K_{th}\big]_{R=\frac{100}{150}} = 5(1-100/150)^{1-0.3} \approx 2.3 MPa$

Thus $\Delta K_I\big]_A > \Delta K_{th}\big]_{R=0.67}$; crack will grow

(b) $\Delta K_{th}\big]_{R=0.67} = 2.3 MPa = (S_{max}-S_{min})\sqrt{\pi(.0025} = (S_{max}-100)\sqrt{\pi(.0025}$

$\boxed{S_{max} = 126 MPa}$

(c) crack length at B

$A''\big]_{A\to B^-} = A/(1-R)^{n(1-\lambda)} = 6.9\times10^{-12}/(1-.67)^{3(1-.5)} = 3.64\times10^{-11}$

$$N = 70\,000 = \frac{2}{(3-2)(3.64\times10^{-11})(150-100)^3\,\pi^{3/2}}\left[\frac{1}{(.0025)^{.5}} - \frac{1}{(a_B)^{.5}}\right]$$

$\frac{1}{a_B^{.5}} = -.8868 + \frac{1}{(.0025)^{.5}}$; $a_B = 0.00274 m = 2.74 mm$

$\boxed{2a_B = 5.48\ mm}$

crack length at C

$A''\big]_{B^+\text{-}C^-} = 6.9\times10^{-12}/(1-100/200)^{3(1-.5)} = 1.95\times10^{-11}$

Problem 6.17 continued

$$N = 30000 = \frac{2}{(3-2)(1.95\times10^{-11})(200-100)^3\pi^{3/2}}\left[\frac{1}{(.00274)^{.5}} - \frac{1}{a_c^{.5}}\right]$$

$$\frac{1}{a_c^{.5}} = -1.6287 + \frac{1}{(.00274)^{.5}} \; ; \quad a_c = 0.00327\,m = 3.27\,mm$$

$$\boxed{2a_c = 6.54\,mm}$$

Crack length at D

$$A''\Big]_{C^+ \cdot D} = 6.9\times10^{-12} \Big/ \left(1 - \frac{100}{250}\right)^{3(1-.5)} = 1.49\times10^{-11}$$

$$N = 10000 = \frac{2}{(3-2)(1.49\times10^{-11})(250-100)^3\pi^{3/2}}\left[\frac{1}{(.00327)^{.5}} - \frac{1}{a_D^{.5}}\right]$$

$$\frac{1}{a_D^{.5}} = -1.400 + \frac{1}{(.00327)^{.5}} \; ; \quad a_D = 0.00386\,m = 3.86\,mm$$

$$\boxed{2a_D = 7.72\,mm}$$

(d)

$$\left.\begin{aligned} K_{I\,B^-} &= S\sqrt{\pi a} = 150\sqrt{\pi(.00274)} = \boxed{13.9\,MPa\sqrt{m}} \\ K_{I\,B^+} &= 200\sqrt{\pi(.00274)} = \boxed{18.6\,MPa\sqrt{m}} \end{aligned}\right\} *$$

$$\left.\begin{aligned} K_{I\,C^-} &= 200\sqrt{\pi(0.00327)} = \boxed{20.3\,MPa\sqrt{m}} \\ K_{I\,C^+} &= 250\sqrt{\pi(0.00327)} = \boxed{25.3\,MPa\sqrt{m}} \end{aligned}\right\} *$$

$$K_{I\,D} = 250\sqrt{\pi(.00386)} = \boxed{27.5\,MPa\sqrt{m}}$$

* B^-, C^- is before load increase, $B^+; C^+$ is after load increase

Problem 6.17 continued

(e) For plane stress $2r_y = \frac{1}{\pi}(K/S_y)^2$

At A: $2r_y = \frac{1}{\pi}(13.3/350)^2 = \boxed{0.46\text{ mm}}$

At B⁺: $2r_y = \frac{1}{\pi}(18.6/350)^2 = \boxed{0.90\text{ mm}}$

At C⁺: $2r_y = \frac{1}{\pi}(25.3/350)^2 = \boxed{1.66\text{ mm}}$

At D: $2r_y = \frac{1}{\pi}(27.5/350)^2 = \boxed{1.96\text{ mm}}$

(f) $a_f = (K_c/S_o)^2/\pi = (110/250)^2/\pi = 0.0616\text{m} = 61.6\text{ mm}$

$$N_f = \frac{2}{(3-2)(1.49\times10^{-11})(250-100)^3\pi^{3/2}}\left[\frac{1}{(.00386)^{.5}} - \frac{1}{(.0616)^{.5}}\right]$$

$N_f = (7{,}142)(12.07) = 86\,209 \approx 86\,000$ cycles

$N_{TOTAL} = 70\,000 + 30\,000 + 10\,000 + 86\,000$

$\boxed{N_{TOT} = 196\,000 \text{ cycles}}$

(g) $S \leq 0.8\,S_y$ for all stress levels. From (e) above,

$r_y(A) = 0.23\text{mm}$ $r_y(C) = 0.83\text{mm}$

$r_y(B) = 0.45\text{mm}$ $r_y(D) = 0.98\text{ mm}$

For cyclic loading $r_y \lessapprox \frac{1}{4}a$ for valid LEFM. This is satisfied. At fracture, $r_y = 15.7\text{mm}$ and $a_f = 61.6\text{mm}$. Thus $r_y \approx \frac{1}{4}a$ at fracture. Therefore LEFM is very reasonable.

For crack length comparison to problem 6.13

Time	Problem 6.13 (2a - mm)	Problem 6.17 (2a - mm)
B	8.36	5.48

Problem 6.17 continued

In problem 6.13, there was more crack growth between time A and time D due to the lower stress ratio observed ($R=0$ and hence higher ΔK). This lead to larger crack lengths at B, C, and D. Also, a larger portion of the total life (N_T) was expended up to time D. In this problem (6.17), because of the higher minimum stress level (100 MPa versus 0 MPa) and lower stress ranges (ΔS's), crack growth was much less between time A and D. This lead to a much greater portion of the total fatigue crack growth life (N_T) occuring after time D. This shows the importance of meanstress and stress range on FCG.