ME 461 – Fatigue and Fracture
Homework 5

Due:
Fri., April 10 (on-campus students)
Fri., Apr. 24 (outreach students)

1. Problem 1, page 239.
2. Problem 8, page 239.
3. Problem 13 parts a) and f), page 240.
4. Problem 17, page 241.
5. Problem 19, page 241.

(All problems are from the textbook)

Problem 7.1

$$\frac{\sigma_y}{S} = 1 + 0.5\left(\frac{r}{x}\right)^2 + 1.5\left(\frac{r}{x}\right)^4 = 1 + 0.5\left(\frac{1}{2}\right)^2 + 1.5\left(\frac{1}{2}\right)^4 = 1.219$$

$$\frac{\sigma_x}{S} = 1.5\left(\frac{r}{x}\right)^2 - 1.5\left(\frac{r}{x}\right)^4 = 1.5\left(\frac{1}{2}\right)^2 - 1.5\left(\frac{1}{2}\right)^4 = 0.281$$

or $\sigma_y = 1.219\,S$ and $\sigma_x = 0.281\,S$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_x = \frac{1.219\,S}{70000} - \frac{0.33(0.281\,S)}{70000} = 0.002$$

or $S = 124$ MPa

If uniaxial stress state is assumed:

$$\epsilon_y = \frac{\sigma_y}{E} \quad \text{or} \quad 0.002 = \frac{1.219\,S}{70000}$$

resulting in $S = 115$ MPa

By assuming a uniaxial stress state, the error is:

$$\frac{124 - 115}{124} \times 100 = 7\%$$

Note that this error is on the nonconservative side as the nominal stress, S, is underestimated by assuming a uniaxial stress state.

problem 7.8

$S_u = 700$ MPa

$K_t = 1.9$ (from prob. 7.3)

$S_f/k_f = 32M_a/\pi d^3$, $S_f \approx 0.5 S_u = 0.5(700) = 350$ MPa

The fatigue limit for bending decreases by a factor of about 0.7 to 0.8 as the diameter increases. Here we use a factor of 0.75, such that $S_f = 0.75(350) = 263$ MPa.

peterson's equation: $k_f = 1 + \frac{k_t - 1}{1 + \frac{a}{r}}$

$$a = 0.0254 \left(\frac{2070}{S_u}\right)^{1.8} = 0.0254 \left(\frac{2070}{700}\right)^{1.8} = 0.179 \text{ mm}$$

$$\text{then } k_f = 1 + \frac{1.9 - 1}{1 + \frac{0.179}{2}} = 1.83$$

If we use Neuber's equation, $k_f = 1 + \frac{k_t - 1}{1 + \sqrt{\rho/r}}$

$\sqrt{\rho} = 0.32 \sqrt{mm}$ from Fig. 7.7.

$$\text{then } k_f = 1 + \frac{1.9 - 1}{1 + \frac{0.32}{\sqrt{2}}} = 1.73$$

Based on peterson's equation, $\frac{S_f}{k_f} = \frac{263}{1.83} = 144$ MPa

$$144 = \frac{32 M_a}{\pi (50)^3} \quad \text{or} \quad M_a = 1.77 \text{ kN·m}$$

Based on Neuber's equation, $\frac{S_f}{k_f} = \frac{263}{1.73} = 152$ MPa

Problem 7.13

$D/d = 50/40 = 1.25$, $\frac{r}{d} = \frac{2}{40} = 0.05$

From Fig. 7.3(a), $k_t = 2.2$

S_f for steels in bending is about $0.5\,S_u$ for $S_u \leq 1400$ MPa and about 700 MPa for $S_u > 1400$ MPa. For 2024-T3 and 7075-T6 aluminum alloys, S_f is given in Table A.1. For 5456-H3 aluminum, S_f can be approximated from stress-life equation and properties listed in Table A.2:

$$S_f \simeq \sigma_f' (2N_f)^b = 826\,(2\times10^8)^{-0.115} = 92 \text{ MPa}$$

S_f values for bending should be reduced by 10 to 25 percent to be used for axial loading. Here we use $(S_f)_{axial} \simeq 0.85 (S_u)_{bending}$

K_f can be calculated from Petersen's equation:

$$k_f = 1 + \frac{k_t - 1}{1 + a/r}, \text{ with } a = 0.0254\left(\frac{2070}{S_u}\right)^{1.8} \text{ for}$$

steels and $a \simeq 0.635$ mm for aluminum alloys.

Knowing S_f and k_f, S_f/k_f could then be calculated, from which fully reversed alternating force P_a can be calculated from:

$$P_a = (S_f/k_f)A, \text{ where } A = \frac{\pi}{4}(40)^2 = 1257 \text{ mm}^2.$$

The results are summarized in the following Table;

Material	Su (MPa)	Sf (MPa)	a (mm)	Kf	Sf/Kf (MPa)	Pa (kN)
RQC-100	931	396	0.107	2.14	185	232
4340	827	351	0.132	2.13	165	208
4340	1240	527	0.064	2.16	244	306
4340	1468	595	0.047	2.17	274	344
4142	1930	595	0.029	2.18	273	343
2024-T3	482	117	0.635	1.91	61	77
5456-H3	400	92	0.635	1.91	48	60
7075-T6	572	134	0.635	1.91	70	88

To find P_a if proper compressive residual stresses are present, we need to construct the Haigh diagram for each material. The properties needed are listed below (from Table A.2). Note that $S_{cat} \approx 30$ MPa for mild steel

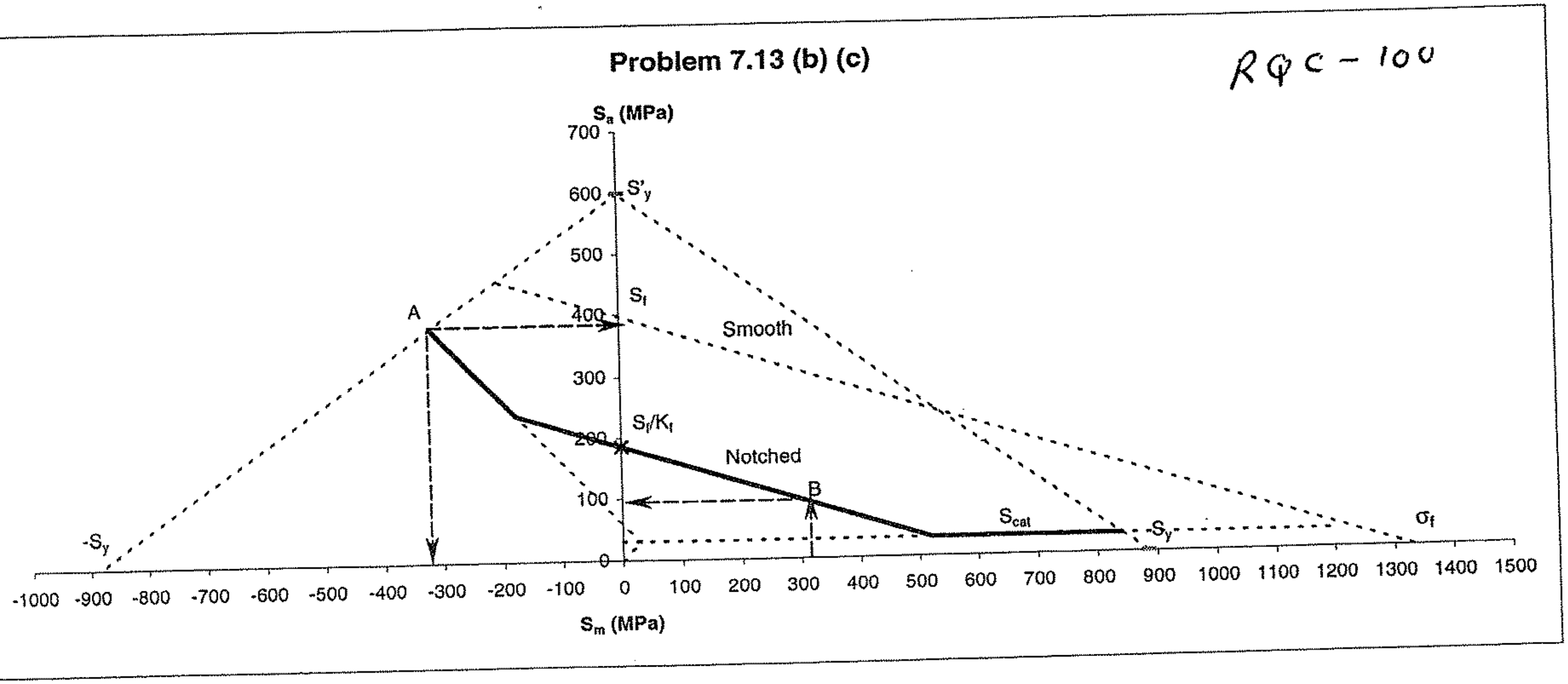
≈ 70 MPa for hard steel

≈ 20 MPa for high strength aluminum.

Material	S_y (MPa)	S_y' (MPa)	σ_f (MPa)	S_{cat} (MPa)	S_f (MPa)	S_f/k_f (MPa)
a) RQC-100	883	600	1330	30	396	185
b) 4340 (S_u=827)	634	471*	1172**	30	351	165
c) 4340 (S_u=1240)	1178	805*	1585**	70	527	244
d) 4340 (S_u = 1468)	1371	863*	1813**	70	595	274
e) 4142	1722	1338*	2275**	70	595	273
f) 2024-T3	379	427	558	20	117	61
g) 5456-H3	234	332*	745**	20	92	48
h) 7075-T6	469	524	745	20	134	70

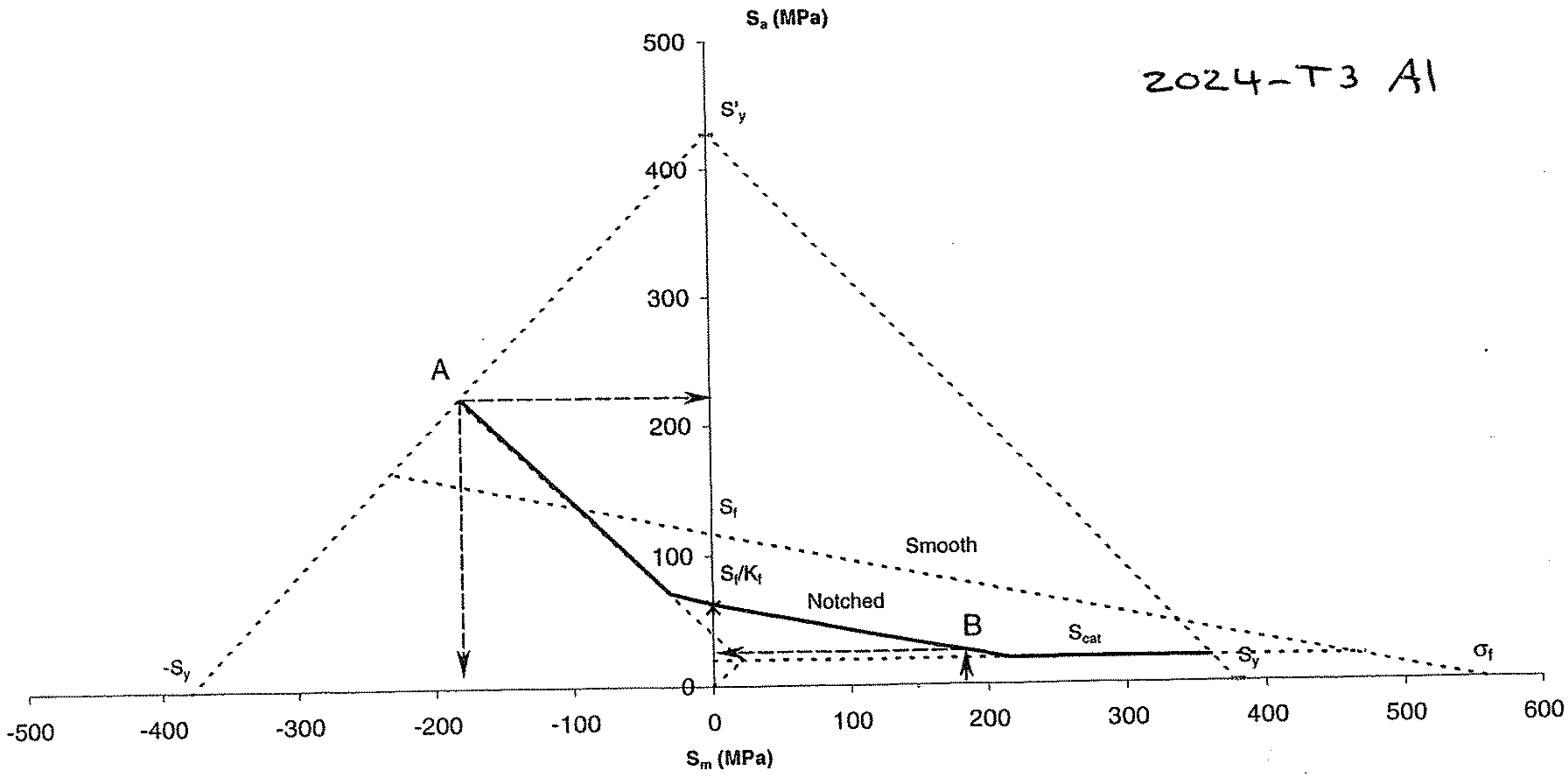
* For this material S_y' is estimated from:
$S_y' \simeq k' (0.002)^{n'}$, where k' and n' are given in Table A.2.

** For the purpose of constructing Haigh diagram, exact value of σ_f is not needed, as the diagram is not very sensitive to its value. Since σ_f is not listed in Table A.2 for this material, we use Eq. 5.20 to approximate it as $\sigma_f \simeq S_u + 345$. This equation is for steels, but here we assume it is reasonable for 5456-H3 Al too. The Haigh diagram for each material is constructed as shown.

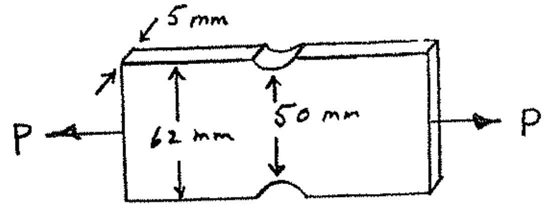


Problem 7.13 (b) (c)
RQC - 100
Sa (MPa)
700
600
500
400
300
200
100
0
S'y
Sf
Smooth
Sf/Kf
Notched
A
B
Scat
-Sy
-Sy
σf
-1000 -900 -800 -700 -600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500
Sm (MPa)

Problem 7.13 (b) (c)



problem 7.17



$\epsilon = 0.002$ at 40 kN

$\epsilon = 0.0025$ at yielding

$\epsilon = 0.0065$ at 80 kN

At 0.002 notch strain, notch behavior is elastic and $\sigma = \epsilon E = (0.002)(200000) = 400$ MPa

$S_{net} = P/A = 40000/(50)(5) = 160$ MPa

then, $k_t = \sigma / S_{net} = 400/160 = 2.5$

At 80 kN force, notch behavior is inelastic since notch strain is larger than 0.002. Nominal net section behavior, however, is elastic since

$$e = \frac{S}{E} = \frac{(80000)/(5)(50)}{200000} = \frac{320}{200000} = 0.0016 < 0.0025$$

$$k_\epsilon = \frac{\epsilon}{e} = \frac{0.0065}{0.0016} = 4.1$$ This is actual (measured) value.

Linear rule: $k_\epsilon = k_t = 2.5$

Neuber's rule: $\epsilon\sigma = (k_t S)^2/E = (2.5 \times 320)^2/200000 = 3.2$

Solve with $\epsilon = \frac{\sigma}{200000} + \left(\frac{\sigma}{950}\right)^{\frac{1}{0.1}} \Rightarrow \epsilon = 0.0060$

$$k_\epsilon = \frac{0.006}{0.0016} = 3.75$$

Strain energy density rule:

$$\frac{\sigma^2}{200000} + \frac{2\sigma}{0.1+1}\left(\frac{\sigma}{950}\right)^{\frac{1}{0.1}} = \frac{(2.5 \times 320)^2}{200000} = 3.2$$

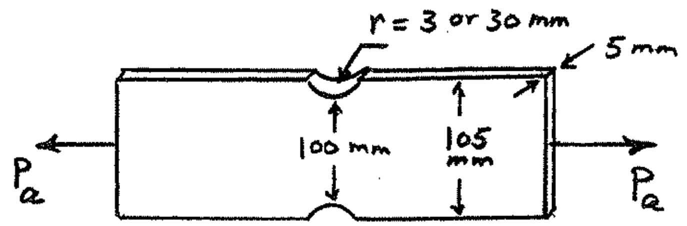
This results in $\sigma = 511$ MPa

then $$\epsilon = \frac{511}{200000} + \left(\frac{511}{950}\right)^{\frac{1}{0.1}} = 0.0046$$

$$K_\epsilon = \frac{0.0046}{0.0016} = 2.86$$

As can be seen from these results, K_ϵ based on Neuber's rule is closer to the measured K_ϵ. Also, K_ϵ based on Glinka's rule falls between predictions from the linear rule and Neuber's rule.

Problem 7.19



RQC-100 steel sheet subjected to fully reversed force, $P_a = 220$ kN.

a) For $r = 3$ mm, using ϵ–N approach determine life to appearance of small crack ≈ 1 mm long.

b) Determine life to fracture

c) Compare (a) and (b) and discuss significance of difference

d) Repeat parts (a) thru (c) for $r = 30$ mm.

Material properties are listed in Table A.2.

a) $$\epsilon_a = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c = \frac{1240}{207000}(2N_f)^{-0.07} + 0.66(2N_f)^{-0.69}$$

This is a thin plate and Neuber's rule with k_f for cyclic loading is reasonable.

$$\frac{H}{d} = \frac{105}{100} = 1.05, \quad \frac{r}{d} = \frac{3}{100} = 0.03$$

from Fig. 7.4 (a), $k_t = 2.75$

Using Peterson's equation, $a = 0.0254\left(\frac{2070}{931}\right)^{1.8} = 0.107$ mm

$$k_f = 1 + \frac{k_t - 1}{1 + a/r} = 1 + \frac{2.75 - 1}{1 + \frac{0.107}{3}} = 2.69$$

$$S_a = \frac{P_a}{A} = \frac{220 \times 10^3}{(100)(5)} = 440 \text{ MPa} \quad \text{and} \quad S_{max} = S_a = 440 \text{ MPa}$$

note that $S_{max} = 440 \text{ MPa} < \approx 0.8 S_y' = 0.8(600) = 480$ MPa.

Therefore, the nominal behavior is elastic.

At the root of the notch however, $K_t S_{max} = (2.75)(440) = 1210\ MPa > S_y'$ and therefore the notch behavior is inelastic.

$$\Delta\sigma\,\Delta\epsilon = \frac{(K_f\,\Delta S)^2}{E} = \frac{(2.69 \times 880)^2}{207000} = 27.1$$

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} = \frac{\Delta\sigma}{207000} + 2\left(\frac{\Delta\sigma}{2 \times 1434}\right)^{\frac{1}{0.14}}$$

resulting in $\Delta\sigma = 1408\ MPa$, $\Delta\epsilon = 0.0192$

Therefore, $\epsilon_a = \Delta\epsilon/2 = 0.0096$. Note that the mean stress is zero since the loading is completely reversed.

$$0.0096 = \frac{1240}{207000}(2N_f)^{-0.07} + 0.66\,(2N_f)^{-0.69} \quad \text{or } N_f = 470 = N_{in}$$

b) First estimate the transitional crack length, l', to check for small crack growth effect. Equation 7.33 in the text is based on Eq. 7.32 for a central crack in a wide plate. For other geometries, the geometry correction factor, α, and consideration of S_{net} & S_{gross} needs to be added to Eq. 7.32 yielding:

$$l' = \frac{c}{\left[\frac{1.12\,K_t}{\alpha}\,\frac{A_{gross}}{A_{net}}\right]^2 - 1}$$

For the double-notched plate given, $2a/w = 5/105 \approx 0.05$ resulting in $Y = 2$ from Fig. 6.3(c) and $\alpha = Y/\sqrt{\pi} = 1.12$.

then,
$$l' = \frac{2.5}{\left[\frac{(1.12)(2.75)}{1.12}\left(\frac{105}{100}\right)\right]^2 - 1} = 0.34\ mm < 1\ mm$$

This mean the small crack effect can be neglected and $a_i = 2.5\ mm + 1\ mm = 3.5\ mm$. Crack growth equation for RQC-100 steel is given in the example problem in Section 7.5 as:

$da/dN = 2.8 \times 10^{-12} (\Delta K)^{3.25}$ for SI units.

K_c is also given to be 165 $MPa\sqrt{m}$ for 9.5 mm thickness. The K_c value for the 5 mm thickness in this problem will be larger, but we'll assume the 165 $MPa\sqrt{m}$ as a conservative estimate.

We first find crack length at fracture from:

$$K_c = Y \frac{P}{BW}\sqrt{a_f}$$, where Y is given in Fig. 6.3 (c).

let $Y \cong 2$, then $165 = 2 \frac{220000}{(5)(105)}\sqrt{a_f}$

or $a_f = 0.0388\ m = 39\ mm$ and $2a = 78\ mm$.

This is too large compared to the available width.

Therefore, calculate $2a_f$ at net section yielding:

$$S_{net} = S_y' = \frac{P_{max}}{A_{net}} = \frac{220000}{(5)(105-2a_f)} = 600$$

or $a_f = 16\ mm$

$a_f/w = 16/105 = 0.15$ and $Y \cong 2$ from a_i to a_f.

Therefore, $$N_f = \frac{a_f^{-\frac{n}{2}+1} - a_i^{-\frac{n}{2}+1}}{(-\frac{n}{2}+1) A (\Delta S)^n Y^n}$$

where $\Delta S = S_{max} - 0 = 220000/(5)(105) = 419\ MPa$

$$N_f = \frac{(0.016)^{-\frac{3.25}{2}+1} - (0.0035)^{-\frac{3.25}{2}+1}}{(-\frac{3.25}{2}+1)(2.8\times10^{-12})(419)^{3.25}(2)^{3.25}} = 3800 \text{ cycles} = N_g$$

c) Comparing crack nucleation life in part (a) of 470 cycles to crack propagation life in part (b) of 3800 cycles, we can conclude that fatigue behavior is dominated by crack growth.

d) part (a) for $r = 30$ mm:

$$\frac{r}{d} = \frac{30}{100} = 0.3, \quad \text{Fig. 7.4 (a) gives } k_t = 1.5$$

then $$k_f = 1 + \frac{1.5 - 1}{1 + \frac{0.107}{30}} \simeq 1.5$$

Note that notch behavior is still inelastic since
$K_t S_{max} = (1.5)(440) = 660 \text{ MPa} > S_y' = 600 \text{ MPa}$.

similar to the solution for part (a):

$$\Delta\sigma\,\Delta\epsilon = \frac{(1.5 \times 880)^2}{207000} = 8.42$$

$$\Delta\epsilon = \frac{\Delta\sigma}{207000} + 2\left(\frac{\Delta\sigma}{2 \times 1434}\right)^{\frac{1}{0.14}}$$

$\Rightarrow$ $\Delta\sigma = 1108$ MPa, $\Delta\epsilon = 0.0076$

Then $\epsilon_a = 0.0038$ and

$$0.0038 = \frac{1240}{207000}(2N_f)^{-0.07} + 0.66\,(2N_f)^{-0.69} \Rightarrow N_f = 9000 = N_n$$

part (b) for $r = 30$ mm remains the same as that for $r = 3$ mm with $N_g = 3800$ cycles, except that the small crack effect is more significant since

$$l' = \frac{2.5}{\left[\frac{(1.12)(1.5)}{1.12}\left(\frac{105}{100}\right) - 1\right]} = 1.2 \text{ mm}$$ is greater than

the initial crack length of 1mm.

We see that when r=30 mm, the fatigue life is now dominated by crack nucleation. Also, the total life for r=30 mm is about 13000 cycles, whereas for r=3 mm, it is about 4000 cycles.