ME 461 – Fatigue and Fracture
Homework 6

Due:
Wed., April 22 (on-campus students)
Wed., May 6 (outreach students)

1. Problem 5, page 268
2. Problem 6, page 269.
3. Problem 9, page 269.

(All problems are from the textbook)

Problem 8.5

From Fig 8.8

S_u - MPa	S_f not peened - MPa	S_f peened - MPa
1000	~ 400	~ 600
1800	~ 350	~ 1050

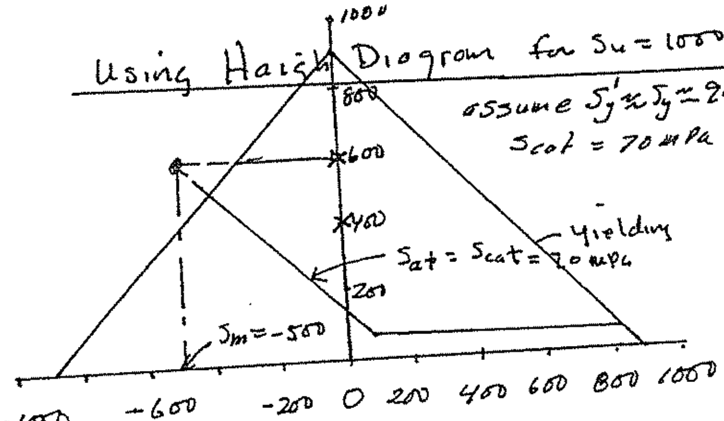
Using modified Goodman $\quad \dfrac{S_a}{S_f} + \dfrac{S_m}{S_u} = 1$

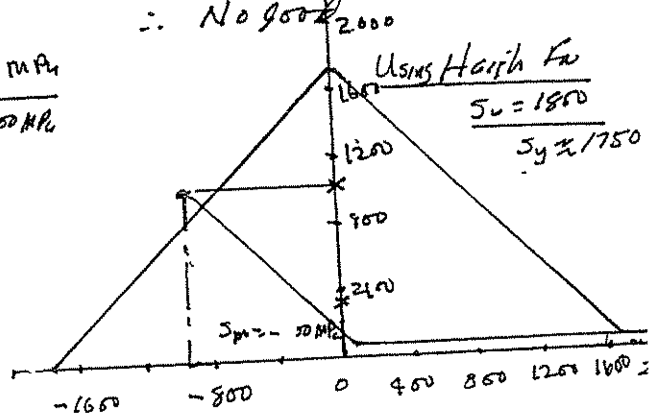
For $S_u = 1000$ MPa $\quad \dfrac{600}{400} + \dfrac{S_m}{1000} = 1 \quad \therefore \boxed{S_m = S_{res} = -500 \text{ MPa}}$

For $S_u = 1800$ MPa $\quad \dfrac{1050}{350} + \dfrac{S_m}{1800} = 1 \quad \therefore \boxed{S_m = S_{res} = -3600 \text{ MPa}}$ *

*but this causes gross yielding ∴ No good

Using Haigh Diagram for S_u = 1000 MPa





based on S_{cat} criterion

$S_m = S_{res} \approx -500$ MPa, but yielding will occur. ∴

$\boxed{S_{res} < -500 \text{ MPa}}$

based on S_{cat} criterion

$S_m \approx S_{res} \approx -950$ MPa but yielding occurs. ∴

$\boxed{S_{res} < -1000 \text{ MPa}}$

From Fig 8.6 $\quad \boxed{\begin{array}{l} S_{res} \leq |-800 \text{ MPa}| \text{ for } S_u = 1000 \text{ MPa} \\ S_{res} \leq |-900 \text{ MPa}| \text{ for } S_u = 1800 \text{ MPa} \end{array}}$

It can be seen that the Haigh diagram provides a very realistic model for estimating the residual stresses in this problem.

Prob 8.6   RQC-100 Steel

$E = 207\ GPa$ $\sigma_f' = 1240\ MPa$ $\epsilon_f' = 0.66$ $b = -0.07$ $c = -0.69$

from Prob 5.8 for smooth surface at

10^3 cycles $\Delta\epsilon/2 = \Delta\epsilon_e/2 + \Delta\epsilon_p/2$

$\boxed{.0070 = .0035 + .0035}$

10^5 cycles $\boxed{.0027 = .0025 + .00015}$

(a) conventional grinding $\sigma_{res} \approx +600\ MPa$. Use Morrow Eq.

$$\Delta\epsilon/2 = \frac{\sigma_f' - \sigma_{res}}{E}(2N_f)^b + \epsilon_f'(2N_f)^c$$

at 10^3 cycles

$$= \frac{1240-600}{207\times10^3}(2\times10^3)^{-.07} + .66(2\times10^3)^{-.69}$$

$$\boxed{.0053 = .0018 + .0035}$$

at 10^5 cycles

$$\Delta\epsilon/2 = \frac{1240-600}{207\times10^3}(2\times10^5)^{-.07} + .66(2\times10^5)^{-.69}$$

$$\boxed{.00147 = .00132 + .00015}$$

(b) gentle grinding $\sigma_{res} \approx -200\ MPa$

at 10^3 cycles

$$\frac{\Delta\epsilon}{2} = \frac{1240-(-200)}{207\times10^3}(2\times10^3)^{-.07} + .66(2\times10^3)^{-.69}$$

$$\boxed{.0076 = .0041 + .0035}$$

at 10^5 cycles

$$\boxed{.00315 = .003 + .00015}$$

Summary	$\Delta\epsilon/2$ at 10^3 cycles	$\Delta\epsilon/2$ at 10^5 cycles
smooth (Prob 5.8)	.0070	.0027
conventional grind	.0053	.0015
gentle grind	.0076	.0031
$\Delta\epsilon/2$ comparison ratio	.0053/.0070 = 0.76 .0076/.0070 = 1.09	.0015/.0027 = .55 .0031/.0027 = 1.15

The comparison ratios indicate significant decrease in allowable $\Delta\epsilon/2$ for conventional grinding (large $+\sigma_{res}$) and small increase for gentle grinding (small $-\sigma_{res}$). Greatest changes at longer life.

Problem 8.9 Assume S_y & S_y' are large enough so that LEFM is applicable.

(a) neglecting weld bead effect, $\Delta K_i = \Delta S \sqrt{\pi a_i} = 200\sqrt{\pi(.001)} = 11.2 \text{ MPa}\sqrt{m}$

this is about Region II

$$\left.\frac{da}{dN}\right]_i = A(\Delta K_i)^n = 10^{-12}(11.2)^3 = \boxed{1.4\times10^{-9} \text{ m/cycle}}$$

(b) $K_T = K + K_{res} = S\sqrt{\pi a} + \sigma_o\sqrt{\pi a}\, f(a/c)$

$$\Delta K_T = \Delta K \qquad R_T = \frac{K_{min} + K_{res}}{K_{max} + K_{res}} = \frac{0 + 14.3}{11.2 + 14.3} = \underline{0.56}$$

where $\left.K_{res}\right]_i = \sigma_o\sqrt{\pi a}\, f(1/2) = 300\sqrt{\pi(.001)}(.85) = 14.3 \text{ MPa}\sqrt{m}$

Use Walker Eq $da/dN = \dfrac{A(\Delta K)^n}{(1-R_T)^{n(1-\lambda)}}$ assume $\lambda \approx 1/2$

$$\left.\frac{da}{dN}\right]_i = \frac{(10^{-12})(11.2)^3}{(1-.56)^{3(1-1/2)}} = \frac{(10^{-12})(11.2)^3}{.29} = \boxed{4.8\times10^{-9} \text{ m/cycle}}$$

(c) We can estimate N_f for both cases.

① No σ_o using Eq. 6.20(g)

$$N_f = \frac{2}{(n-2)A(\Delta S)^n \pi^{n/2}\alpha^n}\left[\frac{1}{a_i^{(n-2)/2}} - \frac{1}{a_f^{(n-2)/2}}\right]$$

$$K_{max} = 200\sqrt{\pi a_f} = K_c = 75 \quad \text{or} \quad a_f = \left(\frac{75}{200}\right)^2\frac{1}{\pi} = 0.0448 \text{ m} = 44.8 \text{ mm}$$

$$N_f = \frac{2}{(3-2)(10^{-12})(200)^3\pi^{3/2}(1)}\left[\frac{1}{\sqrt{.001}} - \frac{1}{\sqrt{.0448}}\right] = \boxed{1.2\times10^6 \text{ cycles}}$$

② $\sigma_o = +300$ MPa

$$N_{total} = N\Big]_{a_i=1\text{mm}}^{a=c=2\text{mm}} + N\Big]_{a=2\text{mm}}^{a_f=44.8\text{mm}}$$

assume $R_T = .56$ stays constant ∴ $A = 10^{-12}/.29$ for $a \le c$

$$N_{total} = \frac{2}{(1)(10^{-12}/.29)(200)^3\pi^{3/2}}\left[\frac{1}{\sqrt{.001}} + \frac{1}{\sqrt{.002}}\right] + \frac{2}{(1)(10^{-12})(200)^3\pi^{3/2}}\left[\frac{1}{\sqrt{.002}} - \frac{1}{\sqrt{.0448}}\right]$$

$$= 1.2\times10^5 + 7.95\times10^5 \approx \boxed{9\times10^5 \text{ cycles}}$$

8.9 cont.

$$\frac{N_f \text{ without } \sigma_o}{N_f \text{ with } \sigma_o} \approx \frac{1.2 \times 10^6}{9 \times 10^5} \approx \boxed{1.3}$$

(d) ① No σ_o

$$\Delta K = \Delta S\sqrt{\pi a_i} \leq \Delta K_{th} \quad \therefore \Delta S = S_{max} = \frac{5}{\sqrt{\pi(.001)}} = \boxed{89 \text{ MPa}}$$

② with $\sigma_o = 300$ MPa $\qquad R_T = 0.56$

from Eq 6.23 $\quad \Delta K_{th}\Big]_{R\neq 0} \approx \Delta K_{th}\Big]_{R=0}(1-R)^{1-\gamma}$

but no γ available. $\therefore$ see Table A.4 where for lower strength steels

$$\Delta K_{th}\Big]_{R=0} \Big/ \Delta K_{th}\Big]_{R\approx 0.6} \approx 2$$

$$\therefore \Delta K_{th}\Big]_{R=.56} \approx 5/2 = 2.5 \text{ MPa}\sqrt{m} = S_{max}\sqrt{\pi(.001)}$$

or $\boxed{S_{max} \approx 44.5 \text{ MPa}}$

The above calculations are reasonable estimates assuming S_y & S_y' are high enough not to cause significant nominal or local yielding so that LEFM is applicable.