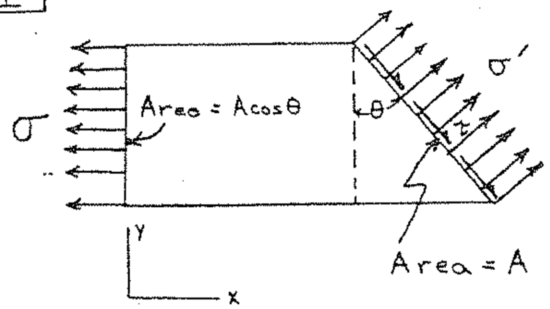
ME 548 – Elasticity

Homework 3

Due: Wed., Oct. 9, 2013

1) Problem 3-3-1 p. 171.
2) Problem 3-3-4 p. 172.
3) Problem 3-3-7 p. 173.
4) Problem 3-3-8 p. 173.
5) Problem 3-4-2 p. 179.
6) Problem 3-4-5 p. 179.

3-3.1



$$\Sigma F_x = \sigma' A\cos\theta + \tau A\sin\theta - \sigma A\cos\theta = 0$$

$$\Sigma F_y = \sigma' A\sin\theta - \tau A\cos\theta = 0$$

$$\therefore\ \tau = \sigma' \frac{\sin\theta}{\cos\theta}$$

$$\sigma'\cos\theta + \sigma' \frac{\sin^2\theta}{\cos\theta} = \sigma\cos\theta$$

or $\quad \sigma' = \sigma\cos^2\theta \qquad\qquad \tau = \sigma\sin\theta\cos\theta$

$\boxed{3-3.4}$

$$\sum \text{Moments} = 0$$

$$\tau \cdot (2\pi \cdot 5) - 20\,000 \cdot 2\pi \cdot 15 = 0 \Rightarrow$$

$$\Rightarrow \tau = 20\,000 \frac{15}{5} = 60{,}000 \text{ psi}$$

3-3.7 stress tensor $= \begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & -5 \\ 4 & -5 & 0 \end{pmatrix}$

plane with normal $N: \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

By Eq (3-3-13), with $n_1 = n_2 = n_3 = \frac{1}{\sqrt{3}}$

$\sigma_\alpha = \sigma_{\beta\alpha} n_\beta = \sigma_{\alpha\beta} n_\beta$ or

$\sigma_{n_1} = \sigma_{11} n_1 + \sigma_{21} n_2 + \sigma_{31} n_3 = \frac{1}{\sqrt{3}}(3+1+4) = \frac{8}{\sqrt{3}} = 4.619$

$\sigma_{n_2} = \sigma_{12} n_1 \; \sigma_{22} n_2 \; \sigma_{32} n_3 = \frac{1}{\sqrt{3}}(1+2-5) = -\frac{2}{\sqrt{3}} = -1.155$

$\sigma_{n_3} = \sigma_{13} n_1 \; \sigma_{23} n_2 \; \sigma_{33} n_3 = \frac{1}{\sqrt{3}}(4-5) = -\frac{1}{\sqrt{3}} = -0.577$

$\sigma_{nn} = \sigma_{11} n_1^2 \; \sigma_{22} n_2^2 + \sigma_{33} n_3^2 + 2\sigma_{12} n_1 n_2 + 2\sigma_{13} n_1 n_3 + 2\sigma_{23} n_2 n_3$

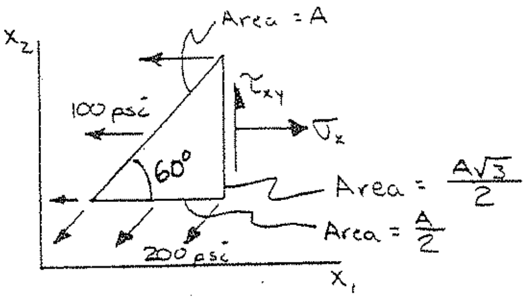
$= \frac{1}{3}(3+2+2+8-10) = \frac{5}{3} = 1.667$

$\sigma_{nt}^2 = \sigma_{n_1} + \sigma_{n_2}^2 + \sigma_{n_3}^2 - \sigma_{nn}^2 = \frac{64+4+1}{3} - \frac{25}{9} = \frac{182}{9}$

$\therefore \sigma_{nt} = \pm \frac{\sqrt{182}}{3} = 4.497$

3-3.8

(a)



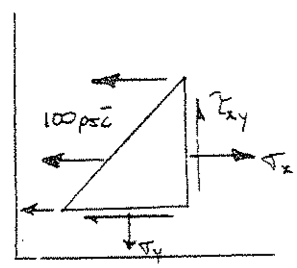
$\Sigma F_{x_1} = \sigma_x \frac{\sqrt{3}}{2} A - 100A - 200\frac{A}{2}\cdot\frac{1}{2} = 0$

$\sigma_x = \frac{2}{\sqrt{3}}(100+50) = 100\sqrt{3}$ psi $= \sigma_{11}$

$\Sigma F_{x_2} = \tau_{xy} A \frac{\sqrt{3}}{2} - 200\frac{A}{2}\cdot\frac{\sqrt{3}}{2} = 0$

$\tau_{xy} = 100$ psi $= \sigma_{12}$

$\sigma_y = 200\frac{\sqrt{3}}{2} = \sigma_{22} = 100\sqrt{3}$ psi



(b) For plane making angles of 45° with the x_1 and x_2 axes, $n_1 = \frac{\sqrt{2}}{2}$, $n_2 = \frac{\sqrt{2}}{2}$, $n_3 = 0$

$\therefore \sigma_{nn} = \sigma_{\alpha\beta} n_\alpha n_\beta = \sigma_{11} n_1^2 + \sigma_{22} n_2^2 + 2\sigma_{12} n_1 n_2$

$= \frac{1}{2}(100\sqrt{3} + 100\sqrt{3} + 200) = 100(1+\sqrt{3})$ psi.

$\boxed{3\text{-}4.2}$ Let $\Sigma_{11} = \sigma_X$, $\Sigma_{22} = \sigma_Y$, $\Sigma_{33} = \sigma_Z$

$\sigma_{11} = \sigma_x$, $\sigma_{22} = \sigma_y$, $\sigma_{33} = \sigma_z$

Then $\Sigma_{11} + \Sigma_{22} + \Sigma_{33} = \sigma_X + \sigma_Y + \sigma_Z$

and $\sigma_{11} + \sigma_{22} + \sigma_{33} = \sigma_x + \sigma_y + \sigma_z$ By Eq (3-4.1)

$$\Sigma_{11} = \sigma_{\alpha\beta}\, a_{1\alpha}\, a_{1\beta}$$

$$\Sigma_{22} = \sigma_{\alpha\beta}\, a_{2\alpha}\, a_{2\beta}$$

$$\Sigma_{33} = \sigma_{\alpha\beta}\, a_{3\alpha}\, a_{3\beta}$$

$$\Sigma_{11} + \Sigma_{22} + \Sigma_{33} = \sigma_{\alpha\beta}\left(a_{1\alpha}a_{1\beta} + a_{2\alpha}a_{2\beta} + a_{3\alpha}a_{3\beta}\right)$$

or $\Sigma_{11} + \Sigma_{22} + \Sigma_{33} = \sigma_{\alpha\alpha} = \sigma_{11} + \sigma_{22} + \sigma_{33}$

by Eqs (1-24.1), (1-24.2), and (1-24.3) Q.E.D

$\boxed{3\text{-}4.5}$ The direction cosines between axes x_α and y_α are

$$a_{11} = a_{33} = \frac{\sqrt{3}}{2}, \quad a_{12} = a_{21} = a_{23} = a_{32} = 0$$

$$a_{13} = -\frac{1}{2}, \quad a_{31} = \frac{1}{2}, \quad a_{22} = 1$$

Hence, by $\Sigma_{\gamma\delta} = \sigma_{\alpha\beta}\, a_{\gamma\alpha}\, a_{\delta\beta}$, we have

$$\Sigma_{11} = \sigma_{\alpha\beta}\, a_{1\alpha}\, a_{1\beta} = a_{11}^2 \sigma_{11} + a_{12}^2 \sigma_{22} + a_{13}^2 \sigma_{33} + 2a_{12}a_{13}\sigma_{23}$$

$$+ 2a_{11}a_{13}\sigma_{13} + 2a_{11}a_{12}\sigma_{12}$$

or $\Sigma_{11} = \left(\frac{\sqrt{3}}{2}\right)^2 a + 0 + 0 + 0 + 0 + 0 = \frac{3}{4} a$

Similarly, $\Sigma_{22} = \sigma_{\alpha\beta}\, a_{2\alpha}\, a_{2\beta} = c$

$$\Sigma_{33} = \sigma_{\alpha\beta}\, a_{3\alpha}\, a_{3\beta} = \frac{1}{4} a$$

also $\Sigma_{12} = \sigma_{\alpha\beta}\, a_{1\alpha}\, a_{2\beta} = a_{11}a_{21}\sigma_{11} + a_{12}a_{22}\sigma_{22} + a_{13}a_{23}\sigma_{33}$

$$+ (a_{12}a_{23} + a_{22}a_{13})\sigma_{23} + (a_{11}a_{23} + a_{21}a_{13})\sigma_{13}$$

$$+ (a_{11}a_{22} + a_{21}a_{12})\sigma_{12} \qquad \text{or } \Sigma_{12} = \frac{\sqrt{3}}{2} b$$