ME 548 – Elasticity

Homework 4

Due: Wed., Oct. 16, 2013

1) Problem 3-5-1 p. 183.
2) Problem 3-6-5 p. 189.
3) Problem 3-6-23 p. 192.
4) Problem 3-6-25 p. 193.

3-5.1

(a) The numerical values of the stress invariants are

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33} = 3 + 1 + 2 \Rightarrow I_1 = 6$$

$$I_2 = \sigma_{11}\sigma_{22} + \sigma_{11}\sigma_{33} + \sigma_{22}\sigma_{33} - \sigma_{12}^2 - \sigma_{13}^2 - \sigma_{23}^2$$

$$= 3 + 6 + 2 - 25 - 64 \Rightarrow I_2 = -78$$

$$I_3 = \begin{vmatrix} 3 & 5 & 8 \\ 5 & 1 & 0 \\ 8 & 0 & 2 \end{vmatrix} = \begin{vmatrix} -29 & 5 & 0 \\ 5 & 1 & 0 \\ 8 & 0 & 2 \end{vmatrix} = -2(29 + 25)$$

$\therefore I_3 = -108$

(b) The principal stresses are the roots of

$$F(\sigma) = \begin{vmatrix} 3-\sigma & 5 & 8 \\ 5 & 1-\sigma & 0 \\ 8 & 0 & 2-\sigma \end{vmatrix} = \sigma^3 - 6\sigma^2 - 78\sigma + 108 = 0$$

$$\sigma_1 = 11.83 \quad , \quad \sigma_2 = 1.28 \quad , \quad \sigma_3 = -7.11$$

(c) principal directions

(i) For $\sigma_1 = 11.83$, we have

$$-8.83N_1 + 5N_2 + 8N_3 = 0$$

$$5N_1 - 10.83N_2 = 0 \Rightarrow N_2 = 0.461N_1$$

$$8N_1 - 9.83N_3 = 0 \Rightarrow N_3 = 0.813N_1$$

$$N_1^2 + N_2^2 + N_3^2 = 1$$

$$N_1 = \pm 0.73$$

$$\therefore \hat{N} : (\pm 0.730, \pm 0.337, \pm 0.595)$$

Similarly, for

(ii) $\sigma_2 = 1.28$

$$\hat{L} : (\pm 0.047, \pm 0.841, \mp 0.528)$$

(iii) $\sigma_3 = -7.11$

$$\hat{M} : (\pm 0.685, \mp 0.422, \mp 0.601)$$

3-6.5

(a) The principal stresses are determined from

$$F(\sigma) = \begin{vmatrix} (-10-\sigma) & 0 & -8 \\ 0 & (2-\sigma) & 0 \\ 8 & 0 & 2-\sigma \end{vmatrix} = 0$$

$$\text{or } (2-\sigma)[(\sigma+10)(\sigma-2)-64] = 0$$

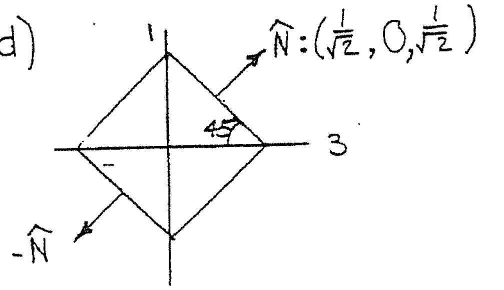
$$\sigma_1 = 6, \quad \sigma_2 = 2, \quad \sigma_3 = -14$$

(b) $9\tau_{oct}^2 = (\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2 = 672$

$$\therefore \tau_{oct} = \frac{4}{3}\sqrt{42} \doteq 8.641$$

(c) $\tau_{max} = \frac{1}{2}|\sigma_1-\sigma_3| = 10 = 1.157\,\tau_{oct}$

(d)



By the theory of Art 3-5, the plane on which τ_{max} acts has direction cosine $(\pm\frac{1}{\sqrt{2}}, 0, \pm\frac{1}{\sqrt{2}})$ relative to principal axes (1, 2, 3). See Table 3-5.1.

3-6.23

(a) with given stresses $I_1 = -40$, $I_2 = -4536$, $I_3 = 0$

$\therefore \sigma^3 + 40\sigma^2 - 4536\sigma = 0$ and

$\sigma = 0$, $\sigma^2 + 40\sigma - 4536 = 0$

The principal stresses are

$\sigma_1 = 50.26$, $\sigma_2 = 0$, $\sigma_3 = -90.26$ MPa

(b) $(\sigma_{nt})_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) = 70.26$ MPa

(c) By Eq (3-6.9),

$9\tau_o^2 = 2I_1^2 - 6I_2 = 2(-40)^2 - 6(-4536) = 30416$

$\therefore \tau_o = 58.13$ MPa

(d) By Eqs (3-5.5) with stresses given and

$\sigma = \sigma_1 = 50.26$, we get

$$-140.26\, a_{11} + 6\, a_{12} = 0$$
$$6\, a_{11} - 0.26\, a_{12} = 0 \qquad (a)$$
$$-50.26\, a_{13} = 0$$

By the last of these equations $a_{13} = 0$

$\therefore a_{11}^2 + a_{12}^2 + a_{13}^2 = a_{11}^2 + a_{12}^2 = 1 \qquad (b)$

By either the first or second of Eqs. (a), cont'd

$a_{12} \approx 23\, a_{11}$

$\therefore a_{11}^2 + (23 a_{11})^2 = 530\, a_{11}^2 \cong 1$

$\therefore a_{11} (= \cos\theta) \cong 0.0434$

and $\theta \approx 87.5°$ counter clockwise from x_1

Since $\sigma_{12} = 6$ is small compared to σ_{11}, σ_{22}, the principal axis for σ_1 is approximately in the x_2 direction.

3-6.25

(a) False

(b) False

(c) True

(d) True

(e) True