Optimality Criteria Problem Set

1) Use the Fitch algorithm to find the minimum cost of the tree shown below:

Answer:
2) Write the likelihood function for the tree and character reconstruction shown below.

Answer:

\[ Pr(R|\tau) = \pi_T \times P_{(T,T)} v_6 \times P_{(T,T)} v_1 \times P_{(T,A)} v_2 \times P_{(T,G)} v_8 \times P_{(G,G)} v_3 \times P_{(G,C)} v_7 \times P_{(C,C)} v_4 \times P_{(C,T)} v_5 \]

3) Suppose you were interested in optimizing the tree below using a 2:1 weighting of transversions over transitions.

a) Construct the step matrix.

```
   --   2   1   2
2   --   2   1
1   2   --   2
2   1   2   --
```
b) Show the optimization on the tree.

Node 1
\[ S_{1(A)} = c_{AC} + c_{AT} = 2 + 2 = 4, \]
\[ S_{1(C)} = c_{CC} + c_{CT} = 0 + 1 = 1, \]
\[ S_{1(G)} = c_{GC} + c_{GT} = 2 + 2 = 4, \]
\[ S_{1(T)} = c_{TC} + c_{TT} = 1 + 0 = 1 \]

Node 2
\[ S_{2(A)} = \min[S_{1A} + c_{Aj}] + c_{AG} \]
\[ = \min[4,3,5,3] + 1 = 3 + 1 = 4 \]
\[ S_{2(C)} = \min[S_{1Cj} + c_{Cj}] + c_{CG} \]
\[ = \min[6,1,6,2] + 2 = 1 + 2 = 3 \]
\[ S_{2(G)} = \min[S_{1Gj} + c_{Gj}] + c_{GG} \]
\[ = \min[5,3,4,3] + 0 = 3 + 0 = 3 \]
\[ S_{2(T)} = \min[S_{1Tj} + c_{Tj}] + c_{TG} \]
\[ = \min[6,2,6,1] + 2 = 1 + 2 = 3 \]