Integration By Parts

Integration by parts is often used when integrating products when a simple substitution does not work. Common integrals of the form $\int x^n e^{kx} dx$, $\int x^n \sin(kx)dx$ or $\int x^n \cos(kx)dx$ are easily integrated by parts. We use the following formula for integration by parts:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Example: $\int 3t^2 e^t dt$

Solution: Let $u = 3t^2$, $dv = e^t dt$. Then by differentiating u we get $du = 6t\,dt$ and by integrating dv we get $v = e^t$.

So, $\int 3t^2 e^t dt = 3t^2 e^t - \int 6te^t dt$. Now we must use integration by parts again on $\int 6te^t dt$.

So, $\int 3t^2 e^t dt = 3t^2 e^t - \left[6te^t - \int 6e^t dt\right]$.

Therefore $\int 3t^2 e^t dt = 3t^2 e^t - 6te^t + 6e^t + C$.

<u>SHORT CUT METHOD:</u>

If you are using integration by parts and the original *u* can by differentiated easily with the nth derivative equal to 0 and if dv is easily integrated, then you can use this short cut. Look at the above example.

We have $u = 3t^2$, $dv = e^t dt$. Notice that eventually the nth derivative of *u* is zero (n = 3) and dv is easily integrated. We can therefore use the short cut method shown below by creating a 3 column table. The first column contains a + or −. We always start with a + and alternate signs down the column. The second column contains the derivatives of *u*. We continue differentiating until the derivative is zero. In the last column, we integrate dv in each entry.

$\pm$	u	dv
+	$3t^2$	e^t
−	$6t$	e^t
+	6	e^t
−	0	e^t

Starting with the first “+” we arrow over to the first u entry and then along the diagonal to the next dv entry. We follow this pattern until we reach the final dv entry. See below:

$\pm$	u	dv
$+$	$3t^2$	e^t
$-$	$6t$	e^t
$+$	6	e^t
$-$	0	e^t

Therefore our result is $\int 3t^2 e^t dt = 3t^2 e^t - 6te^t + 6e^t + C$.

Try looking at some similar problems in a Calculus book and give this a try. It will save you lots of time!!