Math 451, Fall 2012 HW#3 Solutions (chap 3)

(29) There are two ways to interpret the (vague) problem. Here are both solutions.

(a) What is the prob that all 6 balls chosen are unused?

Let $A_0 = \{$ none of the 1st 3 balls are used $\}$

$B = \{$ none of the 2nd 3 balls are used $\}$

$$P(A_0 \cap B) = P(B|A_0)P(A_0)$$

$$= \frac{\binom{6}{3}}{\binom{15}{3}} \cdot \frac{\binom{9}{3}}{\binom{15}{3}} = .0081$$

(b) What is the prob that the last 3 balls chosen are unused?

Let $A_i = \{ i$ of 1st 3 balls are used $\}$, $i = 0, \ldots, 3$

$S = A_0 \cup A_1 \cup A_2 \cup A_3$

So $P(B) = P(B|A_0)P(A_0) + P(B|A_1)P(A_1) + P(B|A_2)P(A_2)$
$+ P(B|A_3)P(A_3)$

$$= \frac{\binom{6}{3}}{\binom{15}{3}} \cdot \frac{\binom{9}{3}}{\binom{15}{3}} + \frac{\binom{8}{3}\binom{6}{1}\binom{9}{2}}{\binom{15}{3}\binom{15}{3}}$$

$$+ \frac{\binom{7}{3}}{\binom{15}{3}} \cdot \frac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} + \frac{\binom{6}{3}}{\binom{15}{3}} \cdot \frac{\binom{6}{3}}{\binom{15}{3}} = .0893$$

(33) Let $R = \{\text{rain}\}$, $E = \{\text{Joe early}\}$ (not early = late)

Know $P(E^c | R) = .3$, $P(E^c | R^c) = .1$, $P(R) = .7$

(a) $P(E) = P(E|R)P(R) + P(E|R^c)P(R^c)$

$= (.7)(.7) + (.9)(.3) = .49 + .27 = .76$

(b) $P(R|E) = \dfrac{P(R \cap E)}{P(E)} = \dfrac{P(E|R)P(R)}{P(E)} = \dfrac{(.7)(.7)}{.76} = \dfrac{.49}{.76}$

$= .6447$

(53) Let $E_i = \{i\text{th component works}\}$, $i = 1, \ldots, n$; $F = \{\text{system functions}\}$

The E_i's are independent and $P(E_i) = 1/2$.

Want $P(E_1 | F) = \dfrac{P(E_1 \cap F)}{P(F)} = \dfrac{P(E_1)}{P(F)}$ since $E_1 \subseteq F$.

Note that $F = \bigcup_{i=1}^{n} E_i$. So

$P(F) = 1 - P(F^c) = 1 - P\left(\left(\bigcup_{i=1}^{n} E_i\right)^c\right)$

$= 1 - P\left(\bigcap_{i=1}^{n} E_i^c\right)$

$= 1 - P(E_1^c)P(E_2^c)\cdots P(E_n^c)$ since

$E_1, \ldots, E_n$ indep

$\Rightarrow E_1^c, \ldots, E_n^c$ indep

$= 1 - \left(\frac{1}{2}\right)^n$.

So $P(E_1 | F) = \dfrac{1/2}{1 - \left(\frac{1}{2}\right)^n} = \dfrac{2^{n-1}}{2^n - 1}$.