Reply to Akiba on the probabilistic measure of coherence

TOMOJI SHOGENJI

Ken Akiba (2000) raises objections to the probabilistic measure of coherence I proposed in Shogenji (1999). According to my proposal, a set \( \{B_1, \ldots, B_N\} \) of beliefs has the following degree \( C(B_1, \ldots, B_N) \) of coherence:

\[
C(B_1, \ldots, B_N) = \text{def.} \frac{\text{Prob}(B_1 \& \ldots \& B_N)}{\text{Prob}(B_1) \times \ldots \times \text{Prob}(B_N)}
\]

Intuitively \( C(B_1, \ldots, B_N) \) measures the degree by which the beliefs are more likely to be true together than they would be if they were related neutrally, i.e. if the truth of one belief had no impact on the truth of any other. It is meant to spell out the frequently proffered characterization that coherent beliefs hang together. Akiba claims my formula is ‘demonstrably inadequate – indeed, inconsistent – for a measure of coherence’ (356) and expresses his general pessimism about the prospect of formulating a plausible probability measure of coherence. To dispel his pessimism I shall address below the three problems Akiba raises.

1. The problem of falsity conduciveness

According to Akiba, coherence as I define it is falsity conducive at the level of individual beliefs in the sense that ‘the other conditions being equal, the more coherent a set is, the less probable the members of the set are’ (357).\(^1\) Akiba finds this ‘very counterintuitive’ but a moment’s reflection reveals that this is exactly the way it should be. When we revise the degree of coherence among beliefs upward, we normally revise the probability of their being all true upward – hence coherence is truth-conducive at the level of sets. However, if for some reason the probability of the beliefs’ being all true should remain the same, we must conclude from the higher degree of coherence that the beliefs are individually less likely to be true than previously thought – hence coherence is ‘falsity-conducive’ at the level of individual beliefs.\(^2\)

\(^1\) That is, if the numerator \( \text{Prob}(B_1 \& \ldots \& B_N) \) in the definition is fixed, an increase in coherence is accompanied by a decrease in the denominator \( \text{Prob}(B_1) \times \ldots \times \text{Prob}(B_N) \).

\(^2\) The point I make here does not affect my discussion of truth conduciveness in the original paper, since the probability of the beliefs’ being all true is not assumed fixed there.
The situation is no different in the case of ordinary empirical justification. When we obtain some empirical evidence in favour of a hypothesis, we normally revise the probability of the hypothesis upward – hence the evidence is truth conducive with regard to the posterior probability. However, if for some reason the probability of the hypothesis should remain the same despite the empirical support, we must conclude that the prior probability should have been lower than we thought – hence the evidence is ‘falsity conducive’ with regard to the prior probability. What we have here is a probabilistic variant of the dictum ‘One philosopher’s modus ponens is another’s modus tollens.’ Whether it is coherence or empirical evidence, one can use any probabilistic support to argue either that the posterior probability should be higher, or that the prior probability should have been lower.

In order to bolster his case against falsity conduciveness Akiba introduces an example of logical entailments involving the following three beliefs:

B₁: The die will come up two.
B₂: The die will come up an even number less than six.
B₂’: The die will come up an even number.

Akiba assumes here that the coherence of {B₁, B₂} should be no different from that of {B₁, B₂’}, since both B₂ and B₂’ are logical consequences of B₁ (357). But the former degree is higher than the latter according to my definition. This is because coherence in my definition is falsity conducive in the sense that given \( \text{Prob}(B₁ & B₂) = \text{Prob}(B₁ & B₂’) \), \{B₁, B₂\} is more coherent than \{B₁, B₂’\} if and only if B₂ is less probable than B₂’. I have already explained why coherence should be falsity conducive in this sense. In the present case this means that contrary to Akiba’s supposition \{B₁, B₂\} should indeed be more coherent than \{B₁, B₂’\}. Akiba is wrong in assuming generally that if A logically entails both B and C, then \{A, B\} should be as coherent as \{A, C\}. To see why this is wrong, consider the following three beliefs:

A: The fossil was deposited 64-to-66 million years ago. (By measurement 1)
B: The fossil was deposited 63-to-67 million years ago. (By measurement 2)
C: The fossil was deposited more than 10 years ago. (By measurement 3)

Although A entails both B and C, there is a clear sense in which A and B hang together more tightly than do A and C, and those familiar with the literature would acknowledge that the concept of coherence in recent epistemology is meant to capture this sense of hanging together. A closer match
between A and B certainly makes us feel more confident about the dating, and most epistemologists think this increase in confidence is justified. I suspect Akiba mistakes the intent of my definition in this regard. I stated in the original article that the satisfactory definition of coherence I seek should capture ‘what epistemologists ordinarily take coherence to be’ (338, emphasis added), and not what logicians, mathematicians or the general public take coherence to be. A satisfactory definition should then assign different degrees of coherence to \{A, B\} and \{A, C\} in accordance with epistemologists’ practice, and my definition passes this test.

2. The problem of self-coherence

Akiba’s misunderstanding of the intent of my definition is more evident in his second objection about self-coherence. Akiba notes that my definition makes the degree of coherence of a single belief always one (357).\(^3\) This means a single belief is neither coherent nor incoherent. Akiba thinks this is a serious flaw, but this is exactly the way it should be in epistemology, where a collection of beliefs is judged coherent when they hang together (are likely to be true together). It makes no sense to say a single belief is coherent or incoherent. Akiba cites beliefs in mathematical and logical truths as examples of self-coherent beliefs, but most epistemologists will call them a priori beliefs.

3. The problem of conjunction

Akiba’s third objection relates to the second but is more substantial. According to my definition, any single belief is neither coherent nor incoherent. This is also true of single conjunctive beliefs. Any single conjunctive belief is neither coherent nor incoherent regardless of its conjuncts. Akiba now argues that for any beliefs B\(_1\) and B\(_2\) the coherence of \{B\(_1\), B\(_2\)\} should be no different from the coherence of \{B\(_1\ \& \ B\(_2\)\}, since a conjunction and its conjuncts are mutually derivable. As a result, my definition makes any pair of beliefs neither coherent nor incoherent, he concludes. This would certainly be ‘the collapse of the whole framework’ (358).

What is missing in Akiba’s reasoning is an individuation of beliefs that is appropriate for the evaluation of coherence. It is common – regrettably, even among epistemologists – to individuate beliefs by their contents, but this will not do for the purpose of evaluating coherence. To see why, we revisit the example of fossil dating with new twists:

A: The fossil was deposited 64-to-66 million years ago. (By measurement 1)

\(^3\) That is, C(B) = \text{Prob}(B)/\text{Prob}(B) = 1 provided \text{Prob}(B) \neq 0. Akiba disregards the non-zero proviso in his discussion of inconsistent beliefs (358).
A*: The fossil was deposited 63-to-67 million years ago. (By inference from A)
B: The fossil was deposited 63-to-67 million years ago. (By measurement 2)
D: The fossil was deposited 64-to-66 million years ago. (By measurement 4)

In this new version, two of the measurements yield exactly the same result in A and D. Consequently, the coherence of A and D is even stronger than that of A and B. Note however that A and D are identical by content-based individuation. We distinguish them nonetheless for the purpose of evaluating coherence because they come from different sources. Meanwhile, despite their difference in contents, A and A* should not be considered distinct when it comes to evaluating coherence, because the latter is derived from the former. It is not surprising that we feel no more confident about the dating when we obtain A* from A by inference. If we wish to relate the concept of coherence to epistemic justification, as most epistemologists do, we must individuate beliefs by their sources, and not by their contents.4 Thus we must reject Akiba’s general claim that the coherence of beliefs should be no different from the coherence of their conjunction. When beliefs have different sources, we cannot treat them as a single conjunctive belief to claim that they are neither coherent nor incoherent.5

References

4 For more on the role of coherence in epistemic justification, and the status of inferential beliefs in particular, see Shogenji (forthcoming).
5 Thanks to Ken Akiba, Tom Lewis and Sheri Smith for their comments on earlier versions. I also benefited from correspondence with Harold Brown and Paul Thagard on the subject.