Stress

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1. Deformation and Stress: Until now, we have focused on the geometry and kinematics of observable structures (i.e., deformation or strain). We now move into a higher level of structural analysis: mechanics (i.e., the origin and relation between forces and stresses and the strain that results).

[Figure. Examples of deformation. (a) Faults in the Cutler Fm near Moab, UT. (b) Deformation bands in hyalotuff, SW Iceland]

2. Force, Stress, and Pressure: Unlike strain, stress cannot be observed. Nor can it be directly measured. Stress is related to force (which has units of Newtons, N); however, it relates to the area over which the force is applied. So stress is:

\[
\sigma = \frac{F}{A} \quad \text{in units of N/m}^2 \quad \text{or} \quad \text{Pa (Pascals)}
\]

In concept, stress is the same as pressure (also measured in Pa); however, pressure is used to refer to fluids (no shear resistance) and is an isotropic quantity, which is typically not true of stress.

[Figure. Same force, different stress]

3. Stress and Rock Strength: A knowledge of stress helps us understand how and when rocks will deform by breaking or flowing. This must happen when some sort of critical stress threshold is reached that defines the strength of a material.

4. Concept of Stress: Stress is a 3D quantity that acts on a rock volume. It is produced by a combination of body forces that act on the entire rock body (e.g., gravity) and surface forces that act on the boundaries of the body (e.g., along the plate boundaries).

[Fig. 4.1. Forces and stresses acting on a volume]

5. Concept of Stress: When considering a 2D surface, forces can be resolved into vectors acting on the surface. The associated stresses (F/A) are also vectors and are referred to as tractions.

Each traction is counter-balanced by an equal and opposite traction acting on the surface (i.e., they are equipollent). These two tractions together define a surface stress.

[Fig. 4.1. Forces and stresses acting on a volume]

6. Concept of Stress: Surface stresses are useful in concept because in structural geology, we are commonly interested on how stresses resolve onto planes (e.g., faults).

For a plane oriented at an angle to the applied force (or stress), we can resolve the stress into two components: the normal stress (\(\sigma_n\)) acting normal to the surface and the shear stress (\(\sigma_s\)) acting parallel to the surface.

[Fig. 4.1. Forces and stresses acting on a volume]
7. **Concept of Stress**: Whereas force vector components resolve simply based on geometry, stresses are more complicated because the area of application depends on the orientation of the plane (given by θ: the angle between σ and the normal to the plane).

\[
\sigma_n = \frac{F_n}{A_\theta} = \frac{(F \cos \theta)}{(A \cos \theta)} = (F/A) \cos 2\theta = \sigma \cos^2 \theta
\]

\[
\sigma_s = \frac{F_s}{A_\theta} = \frac{(F \sin \theta)}{(A \cos \theta)} = (F/A) \sin \theta \cos \theta = \sigma \sin \theta \cos \theta
\]

[Fig. 4.1. Forces and stresses acting on a volume]

8. **Concept of Stress**: \(\sigma_n = \sigma \cos 2\theta\) \(\sigma_s = \sigma \sin \theta \cos \theta\) traction = \(t(n)\)

These equations tell us that the tractions and surface stresses are different on planes with different orientations for the same overall stress state. The normal and shear stresses also differ from the normal and shear forces.

[Fig. 4.2. Distribution of normal and shear forces and stresses acting on a plane of orientation θ]

9. **Stress Sign Convention**: Tractions and surface stresses can be positive or negative in sign, requiring a sign convention.

Tractions are positive if they act in a positive coordinate direction, else they are negative.

Surface stresses consist of a pair of equipollent tractions. Normal stresses are +ive if they are compressive and –ive if they are tensile.

Shear stresses are +ive if they induce CCW rotation (sinistral) and –ive for CW rotation (dextral), when viewed in the negative coordinate direction perpendicular to the shearing direction.

[Box 4.1. Sign conventions for stresses]

10. **The Case of the Curious T-Rex**: Imagine a Tyrannosaurus happily standing on a cube of rock with sides that are 2 m across. The T-Rex exerts a force on the cube (weight, W) of about 18,000 lbs. Converted to Newtons, this is about 80,000 N of force.

The surface stress is \(\sigma = W/A = 80,000 \text{ N} / 2 \text{ m} \times 2 \text{ m} = 20,000 \text{ Pa} = 0.02 \text{ MPa}\)

The normal stress is thus 0.02 MPa and the shear stress is 0 MPa on top of the cube of rock.

[Figure. T-Rex on acid]

11. **The Case of the Curious T-Rex**: Now imagine a surface inside the rock oriented at \(\theta = 30^\circ\). From the transformation equations we derived:
\[ \sigma_n = \sigma \cos^2 \theta = 0.02 \text{ MPa (cos}^2 30^\circ) = 0.015 \text{ MPa} \]

\[ \sigma_s = \sigma \sin \theta \cos \theta = 0.02 \text{ MPa (sin } 30^\circ \text{ cos } 30^\circ) = 0.0087 \text{ MPa} \]

The normal stress is +ive (compressive) but the sign of the shear stress depends on the dip direction of the plane.

Note that this is the case of **uniaxial loading** (from one direction only) and so is not generally applicable in the Earth.

**12. Stress at a Point:** Because the state of stress is different on every plane passing through a point, one plane cannot be used to understand the stress field. In order to fully characterize stress in a rock volume, we need to consider the stresses acting across **all possible plane orientations** passing through that point.

[Figure. **2D representation of 5 surfaces passing through a point. In actuality, there are an infinite number of possible planes**]

**13. Stress at a Point:** As we cannot consider an infinite number of planes, we still need to have some way of thinking about how stress acts on a point. Any one plane will have two tractions defining the **surface stress** that resolves onto that plane from the surrounding stress field.

[Figure. **Trawctions that resolve onto a single plane in 2D**]

**14. Stress at a Point:** If we consider all tractions for all possible planes, and scale them according to their relative magnitudes, they trace out the shape of an ellipse in 2D and an ellipsoid in 3D. These represent the **stress ellipse** (2D) and **stress ellipsoid** (3D).

[Fig. 4.3. **Stress ellipse in 2D and stress ellipsoid in 3D**]

**15. Stress at a Point:** Three **mutually orthogonal** axis lengths are needed to fully characterize an ellipsoid. These are the maximum, intermediate, and minimum principal stresses, respectively (\(\sigma_1, \sigma_2, \text{ and } \sigma_3\)). So \(\sigma_3 \geq \sigma_2 \geq \sigma_1\) always.

The principal stresses are simply three examples of surface stresses out of the infinite number that actually exist. They specifically act on three planes in such a way that they are perpendicular to those planes (i.e., \(\sigma_1, \sigma_2, \text{ and } \sigma_3\) are normal stresses). These are principal planes and they **must** contain zero shear stress.

[Fig. 4.3. **Stress ellipsoid in 3D, showing principal stresses and principal planes**]

**16. Stress at a Point:** Although 3 principal stresses can be used to characterize stress at a point, we can also use the concept of the **stress tensor**, which allows us to characterize stresses in a common coordinate system or frame of reference.

We do this by approximating the point as an **infinitesimally small cube** and considering the **stress components** (normal and shear) that act on the faces of this cube. The cube is oriented such that each of its faces has a **normal vector** that points along one of the defined coordinate axes (\(x, y, \text{ or } z\)).

[Fig. 4.5. **Concept of the stress cube in 3D to approximate the state of stress at a point**]
17. **Stress at a Point**: If the principal stresses happened to act in the x,y,z directions, they would be acting normal to the faces of the cube. The faces of the cube would then be principal planes and there would be no shear stresses on them.

In general, principal stresses may be oriented at some angle to our chosen coordinate system (which may represent N-S, E-W, and up-down). In this case, each face of the cube experiences both normal stress and shear stress.

![Fig. 4.5. Concept of the stress cube in 3D to approximate the state of stress at a point](image)

18. **Stress at a Point**: In fact, each face would experience one normal stress and two shear stresses.

Each component is labeled with a double subscript notation (e.g., \(\sigma_{xy}\)), which respectively represent the direction of the normal to the surface upon which they act, followed by the direction in which they act. This is called the on-in notation.

19. **Stress at a Point**: As this is an infinitesimal small cube, we assume the tractions on one face have equipollent tractions on the opposite face. There are thus 3 sets of faces, each with 3 stress components for a total of 9 stress components. They are:

- \(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}\): 3 normal stresses
- \(\sigma_{xy}, \sigma_{yx}, \sigma_{xz}, \sigma_{zx}, \sigma_{yz}, \sigma_{zy}\): 6 shear stresses

These 9 components fully define the state of stress at a point and indicate that stress is a tensor quantity. A tensor requires magnitudes, directions, and orientation of planes upon which the components act (unlike vectors).

![Fig. 4.5. Concept of the stress cube in 3D to approximate the state of stress at a point](image)

20. **Stress at a Point**: For a stress cube at rest (i.e., not accelerating or rotating), we must also have a balance of stresses to allow equilibrium. This requires that:

- \(\sigma_{xy} = \sigma_{yx}\)
- \(\sigma_{xz} = \sigma_{zx}\)
- \(\sigma_{yz} = \sigma_{zy}\)

There are thus only 6 independent components to any stress tensor.

![Fig. 4.5. Concept of the stress cube in 3D to approximate the state of stress at a point](image)

20. **Stress Tensor Matrix**: The 9 components of a stress tensor are represented in the form of a 3 x 3 matrix, arranged in a specific order:

\[
\begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{bmatrix}
\]

The normal stresses define a diagonal from top left to bottom right.
21. Stress Tensor Matrix: Sometimes, the coordinate axes are labeled as $x_1$, $x_2$, and $x_3$, in which case the stress tensor becomes:

$$
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} \\
\end{bmatrix}
$$