Estimating the Strength of Stabilizing Selection and the Optimum Phenotype

I. Theoretical Background

In class lectures, we have often assumed that a population experiences stabilizing selection such that the fitness of an individual with phenotype z is given by:

$$W = \mathrm{Exp}[-\gamma(z-\theta)^2] \quad (1)$$

where γ is the strength of stabilizing selection and θ is the optimal phenotype.

If stabilizing selection is weak, (1) is approximately equal to:

$$W \approx 1-\gamma(z-\theta)^2 \quad (2)$$

and the change in the population mean phenotype, $\bar{z}$, is given by:

$$\Delta\bar{z} = 2\gamma G(\theta-\bar{z}) \quad (3)$$

In order to predict evolution, we need to estimate both γ and θ.

II. Estimating γ and θ from "real" data

If you have estimated the fitness and phenotype of individuals within a population, it is — in principle — possible to estimate both γ and θ using quadratic regression. Specifically, using the regression model:

$$w = k + \beta z + \gamma z^2 \quad (4)$$

and least squares, any standard statistical package will provide you with estimates for both β and γ. The estimated values of these parameters are the values appearing in evolution equation (2) and *DO NOT NEED TO BE DOUBLED*. In order to estimate θ, simply take the derivative of (4) with respect to z, set this equation equal to zero, and solve for z:

$$\frac{dw}{dz} = \beta + 2\gamma z \quad (5.1)$$

$$0 = \beta + 2\gamma z \quad (5.2)$$

$$z = -\frac{\beta}{2\gamma} = \theta \quad (5.3)$$

Note that some statistical packages such as JMP estimate a slightly different regression equation where the polynomials are centered on the population mean:

$$w = k + \beta z + \gamma(z-\bar{z})^2 \quad (6)$$

In this case, the program will provide you with estimates for β and γ just as discussed in the previous case but you cannot use (5.3) to estimate θ, even though the procedure for identifying θ is identical. Specifically, in order to find θ using the regression equation reported by JMP (or other software that centers the polynomials):

$$\frac{dw}{dz} = \beta + 2\gamma z - 2\gamma\bar{z} \tag{6.1}$$

$$0 = \beta + 2\gamma z - 2\gamma\bar{z} \tag{6.2}$$

$$z = \frac{2\gamma\bar{z} - \beta}{2\gamma} = \theta \tag{6.3}$$