## Mind on Statistics

## Chapter 8

## Sections 8.1-8.2

Questions 1 to 4: For each situation, decide if the random variable described is a discrete random variable or a continuous random variable.

1. Random variable $X=$ the number of letters in a word picked at random out of the dictionary.
A. Discrete random variable
B. Continuous random variable

KEY: A
2. Random variable $X=$ the number of letters in the last name of a student picked at random from a class on English composition.
A. Discrete random variable
B. Continuous random variable

KEY: A
3. Random variable $X=$ the time (in seconds) it takes one email to travel between a sender and receiver.
A. Discrete random variable
B. Continuous random variable

KEY: B
4. Random variable $X=$ the weight (in pounds) a dieter will lose after following a two week weight loss program.
A. Discrete random variable
B. Continuous random variable

KEY: B
5. Which one of these variables is a continuous random variable?
A. The time it takes a randomly selected student to complete an exam.
B. The number of tattoos a randomly selected person has.
C. The number of women taller than 68 inches in a random sample of 5 women.
D. The number of correct guesses on a multiple choice test.

KEY: A
6. Which one of these ' $X$ ' variables is a discrete random variable?
A. An experiment in chemistry is repeated many times and $X$ is the time required for a reaction to occur in seconds.
B. A student is randomly selected and $X$ is the number of correct answers on a two question multiple-choice quiz.
C. A UPS package is randomly selected and $X$ is the weight in pounds of the package.
D. A student is randomly selected and $X$ is the distance they must travel in feet to go from their dorm room door to the door of their first class on Monday morning.
KEY: B

Questions 7 to 10: The probability distribution for $X=$ number of heads in 4 tosses of a fair coin is partially given in the table below.

| $\boldsymbol{k}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X}=\boldsymbol{k})$ | $1 / 16$ | $4 / 16$ | $6 / 16$ | $4 / 16$ | $?$ |

7. What is the probability of getting 4 heads?
A. $1 / 16$
B. $2 / 16$
C. $4 / 16$
D. Cannot be determined

KEY: A
8. What is the probability of getting at least one head?
A. $1 / 16$
B. $4 / 16$
C. $5 / 16$
D. $15 / 16$

KEY: D
9. What is the probability of getting 1 or 2 heads?
A. $4 / 16$
B. $6 / 16$
C. $10 / 16$
D. $14 / 16$

KEY: C
10. What is the value of the cumulative distribution function at 3, i.e. $P(X \leq 3)$ ?
A. $6 / 16$
B. $10 / 16$
C. $11 / 16$
D. $15 / 16$

KEY: D

Questions 11 to 13: Based on her past experience, a professor knows that the probability distribution for $X=$ number of students who come to her office hours on Wednesday is given below.

| $\boldsymbol{k}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X}=\boldsymbol{k})$ | 0.10 | 0.20 | 0.50 | 0.15 | 0.05 |

11. What is the probability that at least 2 students come to office hours on Wednesday?
A. 0.50
B. 0.70
C. 0.80
D. 0.90

KEY: B
12. What is the value of the cumulative probability distribution at 2, i.e. $P(X \leq 2)$ ?
A. 0.50
B. 0.70
C. 0.80
D. 0.90

KEY: C
13. What is the probability that at least 1 student comes to office hours on Wednesday?
A. 0.50
B. 0.70
C. 0.80
D. 0.90

KEY: D

Questions 14 to 17: In an experiment, a person guesses which one of three different cards a researcher has randomly picked (and hidden from the person who guesses). This is repeated four times, replacing the cards each time. Let $X=$ number of correct guesses in the four tries. The probability distribution for $X$, assuming the person is just guessing, is partially provided below.

| No. of Correct Guesses, $\boldsymbol{X}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.20 | 0.39 | 0.30 | 0.10 |  |

14. What is the value of the missing probability $P(X=4)$ ?

KEY: 0.01
15. What is the probability that the person would get 3 or more correct guesses?

KEY: 0.11
16. What is the value of $\mathrm{P}(X \leq 2)=$ probability that number of correct guesses is less than or equal to 2 ?

KEY: 0.89
17. Give the cumulative distribution function for the number of correct guesses.

KEY: See table below.

| No. of Correct Guesses, $\boldsymbol{X}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Cumulative Probability | 0.20 | 0.59 | 0.89 | 0.99 | 1.00 |

Questions 18 and 19: Ellen is taking 4 courses for the semester. She believes that the probability distribution function for $X=$ the number of courses for which she will get an A grade is given below.

| No. of A's, $\boldsymbol{X}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.10 | 0.30 | 0.40 | 0.15 | 0.05 |

18. What is the probability that Ellen will get at least 2 A 's?

KEY: 0.60
19. What is the value of the cumulative probability distribution at 3, i.e. $P(X \leq 3)$ ?

KEY: 0.95

## Section 8.3

Questions 20 to 22: Joan has noticed that the probability distribution for $X=$ number of students in line to use the campus ATM machine when she shows up to use it is shown below.

| $\boldsymbol{k}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{P}(\boldsymbol{X}=\boldsymbol{k})$ | 0.10 | 0.10 | 0.40 | 0.30 | 0.10 |

20. What is the probability that there will be no more than 1 student in line when Joan shows up?
A. 0.10
B. 0.20
C. 0.70
D. 0.90

KEY: B
21. What is the expected value of $X, \mathrm{E}(X)$ ?
A. 2.0
B. 2.2
C. 2.5
D. 3.0

KEY: B
22. The variance of $X, \mathrm{~V}(X)=1.16$. What is the standard deviation of $X$ ?
A. 1.08
B. 1.16
C. 1.35
D. 2.20

KEY: A
23. The payoff $(X)$ for a lottery game has the following probability distribution.

| Payoff, $\boldsymbol{X}$ | $\$ 0$ | $\$ 5$ |
| :--- | :---: | :---: |
| Probability | 0.8 | 0.2 |

What is the expected payoff?
A. $\$ 0$
B. $\$ 0.50$
C. $\$ 1.00$
D. $\$ 2.50$

KEY: C
24. The expected value of a random variable is the
A. value that has the highest probability of occurring.
B. mean value over an infinite number of observations of the variable.
C. largest value that will ever occur.
D. most common value over an infinite number of observations of the variable.

KEY: B
25. In a gambling game, on every play, there is a 0.1 probability that you win $\$ 7$ and a 0.9 probability that you lose
$\$ 1$. What is the expected value of this game?
A. + \$2
B. $\$ 0.20$
C. $-\$ 2$
D. $-\$ 0.20$

## KEY: D

26. Suppose that for $X=$ net amount won or lost in a lottery game, the expected value is $\mathrm{E}(X)=-\$ 0.50$. What is the correct interpretation of this value?
A. The most likely outcome of a single play is a net loss of 50 cents.
B. A player will have a net loss of 50 cents every single time he or she plays this lottery game.
C. Over a large number of plays the average outcome for plays is a net loss of 50 cents.
D. A mistake must have been made because it's impossible for an expected value to be negative

KEY: C
27. The formula for the standard deviation for any discrete random variables with values $x_{i}$ and corresponding probabilities $p_{i}$ is:
A. $\sum x_{i} p_{i}$
B. $\sqrt{\sum\left(x_{i}-\mu\right)^{2} p_{i}}$
C. $\sum\left(x_{i}-\mu\right)^{2} p_{i}$

KEY: B
28. What characteristic of a random variable is described by the expected value?
A. Standard deviation
B. Mean
C. Most likely value
D. Maximum value

KEY: B
29. The following probability distribution is for the random variable $X=$ number of classes for which full time students at a university are enrolled in a semester:

| No. of classes, $\boldsymbol{X}$ | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: |
| Probability | 0.2 | 0.5 | 0.2 | 0.1 |

What is the mean number (expected value) of courses taken per student?
A. 4
B. 5
C. 5.2
D. 5.5

KEY: C
30. The expected value for a random variable is
A. the long-run average.
B. the most likely value.
C. the most frequent value observed in a random sample of observations of the random variable.
D. always $n p$.

KEY: A
31. The mean for a population of $N$ values is equivalent to
A. the mean of any random sample taken from the population.
B. the median of the population of $N$ values.
C. the expected value of a random variable defined as $X=$ value for a randomly sampled individual from the population.
D. $N p$ where $p=$ proportion of values in the population that are unique.

KEY: C

Questions 32 to 34: Ellen is taking 4 courses for the semester. She believes that the probability distribution function for $X=$ the number of courses for which she will get an A grade is given below.

| No. of A's, $\boldsymbol{X}$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.10 | 0.30 | 0.40 | 0.15 | 0.05 |

32. What is the expected number of A's she will get? (i.e. What is $\mathrm{E}(X)$ ?)

KEY: $\mu=\mathrm{E}(X)=0(0.10)+1(0.30)+2(0.40)+3(0.15)+4(0.05)=1.75$ A's.
33. What is the variance for the number of A's she will get? (i.e. What is $\mathrm{V}(X)$ ?)

KEY: $\mathrm{V}(X)=\sigma^{2}=\sum\left(x_{i}-\mu\right)^{2} p_{i}=0.9875$
34. What is the standard deviation for the number of A's she will get?

KEY: $\sigma=\sqrt{\sum\left(x_{i}-\mu\right)^{2} p_{i}}=\sqrt{0.9875}=0.9937$ A's.

Questions 35 to 38: The Southside Bowling Alley has collected data on the number of children that come to birthday parties held at the bowling alley. Let the random variable $X=$ the number of children per party. The distribution for the random variable $X$ is given below.

| Value of $X$ | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.15 | 0.2 | 0.1 | 0.25 | 0.1 | 0.2 |

35. What is the probability that at least 7 children will come to a party?

KEY: $0.1+0.25+0.1+0.2=0.65$
36. Suppose one party is to be randomly selected. We know that there will be at least 7 children at this party. What is the probability that there will be 10 children at the party?
KEY: $0.2 / 0.65=0.308$.
37. Suppose one party is to be randomly selected. What is the expected number of children that will attend this party? Include the appropriate symbol and units in your answer.
KEY: $\mu=\mathrm{E}(X)=5(0.15)+6(0.2)+7(0.1)+8(0.25)+9(0.1)+10(0.2)=7.55$ children.
38. What is the standard deviation for the number of children per party? Include the appropriate symbol and units in your answer.
KEY: $\sigma=\sqrt{\sum\left(x_{i}-\mu\right)^{2} p_{i}}=\sqrt{2.9475}=1.72$ children.

Questions 38 and 40: Did high gas prices keep Americans from hitting the road this past summer? In a nationwide survey of adults, one variable measured was how many days vacationers spent driving on the road on their longest trip. Consider the following (partial) probability distribution for the random variable $X=$ the number of days for the longest car trip.

| Value of $X$ | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :--- | :--- | :--- | :--- |
| Probability | 0.10 | 0.20 | 025 | $?$ | $? ?$ |

39. Suppose the probability of 7 days is twice as likely as the probability of 8 days. What are the two missing probabilities to complete the distribution for $X$ ?
KEY: The total probability provided was 0.55 , leaving 0.45 for the two missing probabilities. With 7 days twice as likely as 8 days, we divide up the remaining 0.45 in a 2 to 1 ratio, thus 0.30 for 7 days and 0.15 for 8 days.
40. What is the expected number of days for the longest trip? Include symbol, value, and units.
$\mathrm{KEY}: \mathrm{E}(X)=\mu=4(0.10)+5(0.20)+6(0.25)+7(0.30)+8(0.15)=0.4+1+1.5+2.1+1.2=6.2$ days.

## Section 8.4

41. Which one of these variables is a binomial random variable?
A. Time it takes a randomly selected student to complete a multiple choice exam.
B. Number of textbooks a randomly selected student bought this term.
C. Number of women taller than 68 inches in a random sample of 5 women.
D. Number of CDs a randomly selected person owns.

## KEY: C

42. Consider an experiment that involves repeatedly rolling a six-sided die. Which of the following is a binomial random variable?
A. The number of rolls until a " 4 " is rolled for the first time.
B. The number of times that a " 4 " is rolled when the die is rolled six times.
C. The sum of the numbers observed on the first six rolls.
D. It is not possible to have a binomial random variable when rolling a six-sided die because a binomial random variable allows only two possible outcomes, not six.
KEY: B
43. Which of the following is an example of a binomial random variable?
A. The number of games your favorite baseball team will win this coming season.
B. The number of questions you would get correct on a multiple-choice test if you randomly guessed on all questions.
C. The number of siblings a randomly selected student has.
D. The number of coins a randomly selected student is carrying.

KEY: B
44. A medication produces side effects in each user with probability 0.10 and this is independent from one person to the next. If 50 people use the medication, the number who will experience side effects is
A. a binomial random variable.
B. always 5 .
C. always $10 \%$.
D. the value for which the probability distribution function (pdf) has the largest value.

KEY: A
45. The probability is $p=0.80$ that a patient with a certain disease will be successfully treated with a new medical treatment. Suppose that the treatment is used on 40 patients. What is the "expected value" of the number of patients who are successfully treated?
A. 40
B. 20
C. 8
D. 32

KEY: D
46. Sara is a frequent business traveler. For security purposes, $10 \%$ of all people boarding airplanes are randomly selected for additional screening just prior to boarding. Define the random variable $X=$ number of flights Sara completes before being chosen for additional screening. For instance, if she is searched boarding her next flight, then $X=0$. What is the value of $P(X=2)=$ probability Sara completes two flights without screening and then is chosen for additional screening on the next one?
A. $(0.1)^{2}$
B. $(0.9)^{2}$
C. $(0.1)^{2}(0.9)$
D. $(0.9)^{2}(0.1)$

KEY: D
47. A landscaping company claims that $90 \%$ of the trees they plant survive (defined as being still alive one year from planting). If a tree does not survive, the company will replace the tree with a new one. A homeowner will have 5 trees planted in his yard by this landscaping company. Consider these 5 trees to be a random sample of all trees planted by this company. If the company's claim is correct, what is the probability that all 5 of the trees will survive?
A. $(0.1)^{5}$
B. $(0.9)^{5}$
C. $(0.1)+(0.1)+(0.1)+(0.1)+(0.1)=0.5$
D. $(0.9)$

KEY: B

Questions 48 to 51: Suppose that a student needs to buy 6 books for her history course. The number of books that she will be able to find used is a binomial random variable $X$ with $n=6$ and $p=0.30$. In other words, the probability that she will find any given book used is 0.30 , and is independent from one book to the next.
48. What is the probability that she will find exactly 2 used books?
A. 0.060
B. 0.185
C. 0.324
D. 0.600

KEY: C
49. What is the probability that she will find exactly 3 used books?
A. 0.060
B. 0.185
C. 0.324
D. 0.900

KEY: B
50. What is the expected number of used books she will find, $\mathrm{E}(X)$ ?
A. 1.8
B. 2.0
C. 3.0
D. $6 . .0$

KEY: A
51. What is the variance for the number of used books she will find, $\mathrm{V}(X)$ ?
A. 0.54
B. 1.12
C. 1.26
D. 61.80

KEY: C

Questions 52 to 55: A child is observing squirrels in the park and notices that some are black and some are gray. For the next five squirrels she sees, she counts $X=$ the number of black squirrels. Suppose $X$ is a binomial random variable with $n=5$ and $p=0.50$.
52. What is the probability that she will see exactly one black squirrel out of the five?
A. 0.031
B. 0.156
C. 0.313
D. 0.500

KEY: B
53. What is the probability that she will see exactly two black squirrels out of the five?
A. 0.031
B. 0.156
C. 0.313
D. 0.500

KEY: C
54. What is the expected number of black squirrels she will see, $\mathrm{E}(X)$ ?
A. 2.0
B. 2.5
C. 3.0
D. 3.5

KEY: B
55. What is the variance for the number of black squirrels she will see, $\mathrm{V}(X)$ ?
A. 1.00
B. 1.12
C. 1.25
D. 2.50

KEY: C

Questions 56 to 60: In a family with 4 children, the number of children with blue eyes is a binomial random variable $X$ with $n=4$ and $p=.20$.
56. What is the probability that all 4 children will have blue eyes?

KEY: 0.0016
57. What is the probability that exactly 3 children will have blue eyes?

KEY: 0.0256
58. What is the probability that none of the children will have blue eyes?

KEY: 0.4096
59. What is the expected number of children with blue eyes, $\mathrm{E}(X)$ ?

KEY: 0.80
60. What is the variance for the number of children with blue eyes, $\mathrm{V}(X)$ ?

KEY: 0.64

## Section 8.5 - 8.7

61. A random variable cannot be both continuous and
A. discrete.
B. uniform.
C. normal.
D. skewed.

KEY: A
62. Which one of the following probabilities is a cumulative probability?
A. The probability that there are exactly 4 people with Type $O+$ blood in a sample of 10 people.
B. The probability of exactly 3 heads in 6 flips of a coin.
C. The probability that the accumulated annual rainfall in a certain city next year, rounded to the nearest inch, will be 18 inches.
D. The probability that a randomly selected woman's height is 67 inches or less.

KEY: D
63. Keeping in mind that a normal distribution is a model for a continuous variable, which one of the following variables cannot possibly have a normal distribution?
A. People's opinions about a new tax law (favor or oppose)
B. Weights of 5 -year-old male children
C. Handspans of adult females
D. Ounces of soda in cans labeled as having 12 ounces

KEY: A
64. The time taken to answer an exam question for a randomly chosen student has a uniform probability distribution from 1 minute to 5 minutes. What is the probability that the time to answer is no more than 2 minutes?
A. 0.20
B. 0.25
C. 0.40
D. 0.75

KEY: B
65. The time taken to deliver a pizza has a uniform probability distribution from 20 minutes to 60 minutes. What is the probability that the time to deliver a pizza is at least 25 minutes?
A. 0.125
B. 0.300
C. 0.700
D. 0.875

KEY: D

Questions 66 to 68: The time taken for a computer to boot up, $X$, follows a normal distribution with mean 30 seconds and standard deviation 5 seconds.
66. What is the standardized score ( $z$-score) for a boot-up time of $x=30$ seconds?
A. -2.0
B. 0.0
C. 1.0
D. 2.0

KEY: B
67. What is the standardized score ( $z$-score) for a boot-up time of $x=20$ seconds?
A. -2.0
B. 0.0
C. 1.0
D. 2.0

KEY: A
68. What is the standardized score ( $z$-score) for a boot-up of time $x=35$ seconds?
A. -2.0
B. 0.0
C. 1.0
D. 2.0

KEY: C
Questions 69 to 72: Find the requested probability for the standard normal random variable $Z$.
69. What is the probability that $Z$ is less than or equal to $2, P(Z \leq 2)$ ?
A. 0.0228
B. 0.2000
C. 0.5000
D. 0.9772

KEY: D
70. What is the probability that $Z$ is greater than $2, P(Z>2)$ ?
A. 0.0228
B. 0.2000
C. 0.5000
D. 0.9772

KEY: A
71. What is the probability that $Z$ is between -1 and $1, P(-1 \leq Z \leq 1)$ ?
A. 0.1587
B. 0.3174
C. 0.6826
D. 0.8413

KEY: C
72. What is the probability that $Z$ is between -1.2 and $1.45, P(-1.2 \leq Z \leq 1.45)$ ?
A. 0.0303
B. 0.7740
C. 0.8041
D. 0.8114

KEY: D
73. Assuming a standard normal distribution is appropriate, what is the approximate probability that a $z$-score is greater than or equal to 2.33 ? Said another way, what is $P(Z \geq 2.33)$ ?
A. 0.99
B. 0.01
C. 0.15
D. 0.25

KEY: B
74. Suppose that vehicle speeds at an interstate location have a normal distribution with a mean equal to 70 mph and standard deviation equal to 8 mph . What is the z -score for a speed of 64 mph ?
A. -6
B. -0.75
C. +0.75
D. +6

KEY: B
75. Heights of college women have a distribution that can be approximated by a normal curve with a mean of 65 inches and a standard deviation equal to 3 inches. About what proportion of college women are between 65 and 67 inches tall?
A. 0.75
B. 0.50
C. 0.25
D. 0.17

KEY: C
76. Verbal SAT scores have approximately a normal distribution with mean equal to 500 and standard deviation equal to 100 . The $95^{\text {th }}$ percentile of $z$-scores is $z=1.65$. What is the $95^{\text {th }}$ percentile of verbal SAT scores?
A. 335
B. 500
C. 600
D. 665

KEY: D
77. Verbal SAT scores have a mean of 500 and a standard deviation of 100 . Which of the following describes how to find the proportion of Verbal SAT scores that are greater than 600 ?
A. Find the area to the left of $\mathrm{z}=1$ under a standard normal curve.
B. Find the area between $\mathrm{z}=-1$ and $\mathrm{z}=1$ under a standard normal curve.
C. Find the area to the right of $\mathrm{z}=1$ under a standard normal curve.
D. Find the area to the right of $\mathrm{z}=-1$ under a standard normal curve.

KEY: C
78. Pulse rates of adult men are approximately normal with a mean of 70 and a standard deviation of 8 . Which choice correctly describes how to find the proportion of men that have a pulse rate greater than 78 ?
A. Find the area to the left of $z=1$ under a standard normal curve.
B. Find the area between $z=-1$ and $z=1$ under a standard normal curve.
C. Find the area to the right of $z=1$ under a standard normal curve.
D. Find the area to the right of $z=-1$ under a standard normal curve.

## KEY: C

79. Scores on an achievement test had an average of 70 and a standard deviation of 10 . Serena's score was 85 . Assuming the scores have approximately a normal distribution, about what proportion of students scored lower than Serena?
A. 0.93
B. 0.07
C. 0.84
D. 0.68

KEY: A
80. Weights of females have approximately a normal distribution with mean 135 lbs . and standard deviation 20 lbs. Allison weighs 145 lbs . What is the $z$-score for her weight?
A. 10
B. 1.50
C. 0.50
D. 0.20

KEY: C

Questions 81 to 84: The average time taken to complete an exam, $X$, follows a normal probability distribution with mean $=60$ minutes and standard deviation 30 minutes.
81. What is the probability that a randomly chosen student will complete the exam in 30 minutes or less?
A. 0.1587
B. 0.5000
C. 0.8413
D. 0.9772

KEY: A
82. What is the probability that a randomly chosen student will take at least an hour to complete the exam?
A. 0.1587
B. 0.5000
C. 0.8413
D. 0.9772

KEY: B
83. By what time should $67 \%$ of the students have finished the exam? (i.e. What is the $67^{\text {th }}$ percentile for $X$ ?)
A. 60.0 minutes
B. 73.2 minutes
C. 77.9 minutes
D. 82.5 minutes

KEY: B
84. By what time should $99 \%$ of the students have finished the exam? (i.e. What is the $99^{\text {th }}$ percentile for $X$ ?)
A. 85.2 minutes
B. 89.4 minutes
C. 129.9 minutes
D. 150.0 minutes

KEY: C
85. The number of first-year students who will decide to live in dormitories is a binomial random variable $X$ with $n=1000$ and $p=0.50$. Use the normal approximation with continuity correction to approximate the probability that $X$ will be no more than $490, P(X \leq 490)$.
A. 0.2743
B. 0.2709
C. 0.7291
D. 0.7357

KEY: A
86. The normal approximation to the binomial distribution is most useful for finding which of the following?
A. The probability $P(X=k)$ when $X$ is a binomial random variable with small $n$.
B. The probability $P(X \leq k)$ when $X$ is a binomial random variable with large $n$.
C. The probability $P(X=k)$ when $X$ is a normal random variable with small $n$.
D. The probability $P(X \leq k)$ when $X$ is a normal random variable with large $n$.

KEY: B
87. Suppose that in a large population, the proportion that is left-handed is $p=0.10$. Suppose $n=20$ people will be randomly selected and $X=$ number of people in the sample who are left-handed. What probability model should be used to find the probability that $X=3$ ?
A. Binomial
B. Normal
C. Uniform
D. Chi-square

KEY: A
88. Suppose that a quiz consists of 20 True-False questions. A student hasn't studied for the exam and will just randomly guesses at all answers (with True and False equally likely). How would you find the probability that the will student get 8 or fewer answers correct?
A. Find the probability that $X=8$ in a binomial distribution with $n=20$ and $p=0.5$.
B. Find the area between 0 and 8 in a uniform distribution that goes from 0 to 20 .
C. Find the probability that $X=8$ for a normal distribution with mean of 10 and standard deviation of $\sqrt{5}$.
D. Find the cumulative probability for 8 in a binomial distribution with $n=20$ and $p=0.5$.

KEY: D

Questions 89 to 92: Suppose the time to wait for placing an order at a drive-through window has a uniform distribution between 0 and 8 minutes.
89. What proportion of customers is expected to wait more than 6 minutes?

KEY: 0.25 or $25 \%$
90. Complete the sentence: About $25 \%$ of the customers are expected to wait at most $\qquad$ minutes?
KEY: 2
91. What is the expected waiting time?

KEY: $E(X)=4$ minutes
92. What is the interquartile range for waiting time?
$\mathrm{KEY}: \mathrm{IQR}=6-2=4$ minutes.
Questions 93 to 95: Some managers of companies use employee rankings to laud the best and let go of the worst. Suppose the distribution of rankings of employees at a large company is normal with a mean of 65 points and a standard deviation of 6 points.
93. What proportion of employees has a ranking above 59 points?

KEY: The $z$-score is -1 , so the proportion is 0.8413 .
94. Managers at this large company were told to determine the top 20 percent, the bottom 10 percent and the remaining 70 percent in the middle, and then "weed out" (let go) those in that bottom tier. Using the provided model for rankings, what is the cut-off for an employee to be in the top 20 percent?
KEY: The $80^{\text {th }}$ percentile is 70.05 points.
95. Using the provided model for rankings, what is the cut-off for an employee to be "weeded out?"

KEY: The $20^{\text {th }}$ percentile is 57.3 points.
Questions 96 and 97: The time to complete an exam for a randomly chosen student in a textiles class is a normal random variable with a mean of 50 minutes and a standard deviation of 10 minutes.
96. What percent of the class will have finished the exam by 75 minutes?

KEY: $99.38 \%$
97. At what time will $75 \%$ of the class have finished the exam? (i.e. What is the $75^{\text {th }}$ percentile of the distribution?)
KEY: 56.7 minutes

Questions 98 to 100: Suppose $X$ is a binomial random variable with $n=800$ and $p=0.20$.
98. Find the expected value $E(X)$.

KEY: $E(X)=800(0.20)=160$.
99. Find the standard deviation of $X$.

KEY: The standard deviation of $X$ is 11.31 .
100. Find the approximate the probability $P(X \geq 180)$ with the normal approximation with a continuity correction. KEY: 0.0427

Questions 101 to 103: According to the National Education Association, the country's largest teachers union, the proportion of men in teaching is at its lowest level in 40 years. Only 21 percent of teachers in U.S. public schools are men. In early grades, the gender ratio is even more imbalanced - just 9 percent of elementary school teachers are men. Suppose we randomly select 120 teachers who work in U.S. public schools
101. How many men would we expect to find in the sample of 120 teachers?

KEY: 25.2 men
102. What is the standard deviation for the number of men in a sample of 120 teachers?

KEY: 4.46 men
103. What is the (approximate) probability that less than 20 of these teachers are men?

KEY: 0.1218

Questions 104 to 106: Midterm Exam Results reported as $z$-scores.
104. The professor for one section of a multi-section math course will be reporting his students' midterm exam scores as $z$-scores. Jackson's score on the midterm exam was 82 points. The section's class mean was 75 and the standard deviation was 5 points. What was his $z$-score?
$\mathrm{KEY}: \mathrm{Z}=(82-75) / 5=1.4$
105. Greg is in another section of the course. The distribution of midterm scores for his section was reported as being approximately normal with a mean of 62 and a standard deviation of 8 . Greg's standardized score is +1 . What is Greg's midterm score?
KEY: $Z=+1$ implies Greg's score is 1 standard deviation above the mean $=62+1(8)=70$ points
106. Continuation of $\mathbf{1 0 5}$. What percentile does Greg's score correspond to?

KEY: By the empirical rule we could say $16 \%$ will be above 70 points. Thus a score of 70 points corresponds to the $84^{\text {th }}$ percentile. Using a table or calculator to obtain the exact area to the left of a $\mathrm{z}=1$, we could report Greg's score as the $84.13^{\text {th }}$ percentile.

## Section 8.8

Questions 107 and 108: A bank offers free coffee to its customers. The coffee is brewed each morning in a large coffee urn, and the amount brewed varies slightly. It is independent from day to day, with mean of 125 ounces and standard deviation of 3 ounces. The amount customers drink from day to day is also random, with mean of 100 ounces and standard deviation of 5 ounces, and is independent of the amount brewed.
107. What is the mean and standard deviation for the amount of coffee remaining at the end of the day?
A. Mean $=25$ ounces, standard deviation $=\sqrt{5-3}=1.41$ ounces
B. Mean $=25$ ounces, standard deviation $=\sqrt{5+3}=2.83$ ounces
C. Mean $=25$ ounces, standard deviation $=\sqrt{5^{2}-3^{2}}=4$ ounces
D. Mean $=25$ ounces, standard deviation $=\sqrt{5^{2}+3^{2}}=5.83$ ounces

KEY: D
108.If the amount of coffee brewed and the amount of coffee customers drink are both normally distributed, what is the probability that amount of coffee remaining at the end of a day exceeds 40 ounces?
A. 0
B. 0.0051
C. 0.9949
D. 0.4000

KEY: B
Questions 109 to 111: The time it takes a student to finish buying his/her text books, $W$, is a normal random variable. This variable $W$ is the sum of two other normal variables, $X$ and $Y(\mathrm{~W}=X+Y)$, where $X=$ the time to wait in line at the ATM machine to get cash, and $Y=$ the time to wait in line at the cashier to buy the books. Assume that $X$ and $Y$ are independent normal random variables, with the following means and standard deviations:

| Random Variable | Mean (minutes) | Standard Deviation (minutes) |
| :--- | :---: | :---: |
| $X$ (waiting time for ATM machine) | 4 | 5 |
| $Y$ (waiting time for cashier) | 8 | 12 |

109. What is the expected time it takes a student to finish buying his/her text books?
A. 4 minutes
B. 8 minutes
C. 12 minutes
D. 17 minutes

## KEY: C

110. What is the standard deviation for the time it takes a student to finish buying his/her text books?
A. 5 minutes
B. 12 minutes
C. 7 minutes
D. 13 minutes

KEY: D
111. What is the probability that a student has to wait no more than 10 minutes total to buy her books?
A. 0.4404
B. 0.4522
C. 0.5596
D. 0.5948

KEY: A

Questions 112 and 113: Centerpieces are normally distributed with mean of 30 ounces and standard deviation of 2 ounces. The weights of the shipping boxes are normally distributed with mean of 12 ounces and standard deviation of 1 ounce. Suppose that centerpieces are chosen at random and packed into randomly chosen shipping boxes.
112. What are the mean and standard deviation of the total weights of the packages?

KEY: Mean $=42$ ounces, standard deviation $=2.24$ ounces
113. If a package exceeds 45 ounces, an additional charge is incurred. What is the proportion of packages will incur such a charge?
KEY: 0.0901
114. Suppose that $X$ and $Y$ are independent binomial random variables described in the table below.

| Binomial Random Variable | $\boldsymbol{n}$ | $\boldsymbol{p}$ |
| :--- | :---: | :---: |
| $X$ | 5 | 0.50 |
| $Y$ | 2 | 0.50 |

Let $W=X+Y$. Find the probability that $W=4$.
KEY: 0.2734
Questions 115 and 116: John and Mary both leave for class at the same time. The time it takes them to get to class, ( $J=$ time for John, $M=$ time for Mary) are independent normal random variables. For John, the mean time $\mathrm{E}(J)=10$ minutes with variance $\mathrm{V}(J)=4$ minutes. For Mary, the mean time $\mathrm{E}(M)=8$ minutes with variance $\mathrm{V}(M)=1$ minute.
115. What is the probability that Mary will get to class before John does?

KEY: 0.8133
116. What is the probability that John will get to class before Mary does?

KEY: 0.1867

Chapter 8

