1. Prove \( f \ast (g_1 + g_2) = f \ast g_1 + f \ast g_2 \)

\[
f \ast (g_1 + g_2) = \int_0^t f(t - \tau) [g_1(\tau) + g_2(\tau)] \, d\tau
\]

\[
= \int_0^t [f(t - \tau)g_1(\tau) + f(t - \tau)g_2(\tau)] \, d\tau
\]

\[
= \int_0^t f(t - \tau)g_1(\tau) \, d\tau + \int_0^t f(t - \tau)g_2(\tau) \, d\tau
\]

\[
= f \ast g_1 + f \ast g_2
\]

\[
\therefore f \ast (g_1 + g_2) = f \ast g_1 + f \ast g_2
\]