Today's Objectives:
1. Determine the moment of a force about an axis using scalar analysis, and
2. Determine the moment of a force about an axis using vector analysis

Concept
Recall that the moment of a force about any point \( A \) is \( M_A = F \, d_A \)
where \( d_A \) is the perpendicular (or shortest) distance from the point to the force's line of action.
This concept can be extended to find the moment of a force about an axis.

Moment about an axis
Step 1:
Compute the moment of the Force \( F \) about any arbitrary point \( O \) that lies on the a-a axis using the cross product.
\[
M_O = \mathbf{r} \times \mathbf{F}
\]

Step 2:
Find the component of \( M_O \) along the axis a-a using the dot product.
\[
M_a = u_a \cdot M_O
\]

Facts:
Moment of a force about point “O” is a vector
Moment of a force about a line is scalar

In the figure above, the moment about the y-axis would be \( M_y = 20 \times (0.3) = 6 \text{ N} \cdot \text{m} \).
the moment about the x-axis would be \( M_x = 20 \times (0.4) = 8 \text{ N} \cdot \text{m} \).
However this calculation is not always trivial and vector analysis may be preferable.

Moment about an axis
Triple scalar product.
\[
M_a = u_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}
\]

\( u_a \) represents the unit vector along the axis a'-a axis,
\( \mathbf{r} \) is the position vector from any point on the a'-a axis to any point A on the line of action of the force, and
\( \mathbf{F} \) is the force vector.
Example

Given: A force of 80 lb acts along the edge DB.

Find: The magnitude of the moment of this force about the axis AC.

Solution Plan

\[ M_{AC} = u_{AC} \cdot (r_{AB} \times F_{DB}) \]

1) Find \( u_{AC} = \frac{r_{AC}}{r_{AC}} \)
2) Find \( F_{DB} = 80 \text{ lb} \)
3) Find \( u_{DB} = 80 \text{ lb} \)
4) Complete the triple scalar product.

Position vectors

\[ r_{AB} = \{ 20 \text{ j} \} \text{ ft} \]
\[ r_{AC} = \{ 13 \text{ i } + 16 \text{ j} \} \text{ ft} \]
\[ r_{DB} = \{ -5 \text{ i } + 10 \text{ j } – 15 \text{ k} \} \text{ ft} \]

Unit and Force vectors

\[ u_{AC} = \frac{13 \text{ i } + 16 \text{ j}}{13^2 + 16^2} \text{ ft} \]
\[ F_{DB} = 80 \frac{r_{DB}}{(5^2 + 10^2 + 15^2)} \text{ lb} \]

\[ u_{AC} = 0.6306 \text{ i } + 0.7761 \text{ j} \]
\[ F_{DB} = -21.38 \text{ i } + 42.76 \text{ j } – 64.14 \text{ k} \text{ lb} \]

\[ M_{AC} = 0.6306 \{ 20 \text{ (-64.14)} – 0 – 0.7706 \text{ (0 – 0)} \} \text{ lb } \cdot \text{ ft} \]
\[ = -809 \text{ lb } \cdot \text{ ft} \]