Homework 2
MATH 430, Fall 2014

Due: Friday, 12 September, in class.
1) All work must be shown clearly for full credit. You must justify all your answers.
2) You may discuss the problems with other students but solutions must be written independently.

Section 1.4
1. Show that the vectors \((1, 1, 0), (1, 0, 1),\) and \((0, 1, 1)\) span \(\mathbb{R}^3\).

Section 1.5
2. Let \(S = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}\) be a subset of the vector space \(\mathbb{R}^3\).
   Prove that \(S\) is linearly independent.
3. Is the following set linearly dependent or independent?
   \[
   \left\{ \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ -4 & -4 \end{pmatrix} \right\} \in M_{2 \times 2}(\mathbb{R})
   \]
4. Let \(M\) be a square upper triangular matrix with non-zero diagonal entries. Prove that the columns of \(M\) are linearly independent.

Section 1.6
5. Find three different basis for the space \(M_{2 \times 2}(\mathbb{R})\).
6. The vectors \(u_1 = (1, 1, 1, 1), u_2 = (0, 1, 1, 1), u_3 = (0, 0, 1, 1)\) and \(u_4 = (0, 0, 0, 1)\) form a basis for \(\mathbb{R}^4\). Find the unique representation of an arbitrary vector \((a_1, a_2, a_3, a_4)\) in \(\mathbb{R}^4\) as a linear combination of \(u_1, u_2, u_3,\) and \(u_4\).