Homework 9
MATH 430, Fall 2014

Due: Friday, 5 December, in class.
1) All work must be shown clearly for full credit. You must justify all your answers.
2) You are encouraged to discuss the problems with other students but solutions must be written independently.

Section 6.5

1. Let
\[ A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} . \]
Find an orthogonal matrix \( Q \) such that \( Q^*AQ \) is diagonal. Hence find new coordinates \( x', y' \) so that the following quadratic form \( x^2 + 4xy + y^2 \) can be written as \( \lambda_1(x')^2 + \lambda_2(y')^2 \). Specify \( \lambda_1 \) and \( \lambda_2 \).
(Note: the first half of the question was Q. 4 of Midterm II.)

2. Find new coordinates \( x', y' \) so that the following quadratic forms can be written as \( \lambda_1 x'^2 + \lambda_2 y'^2 \).
   (a) \( x^2 - 12xy - 4y^2 \)
   (b) \( x^2 - 2xy + y^2 \)

Section 6.7

3. Find a singular value decomposition for each of the following matrices.
   (a)
   \[ A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \]
   (b)
   \[ A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \]